Search Friction, Liquidity Risk and Bond Misallocation

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Abstract

Systemic search friction is an important liquidity factor which drives all corporate bonds' yield spread changes. In cross section, bonds have different levels of this yield spread loading. To explain this cross-sectional heterogeneity, we propose a measure of bond-level misallocation among traders, which is defined as the covariance of traders' private valuation and inventory position for each bond. Using transaction-level data, we find that: bonds with a higher level of misallocation have a lower *absolute* value of yield spread loading on systemic search friction. This relationship is specific to the decentralized market structure, where transactions rely on traders' searching activity.

Keywords: corporate bond market, bond misallocation, liquidity risk, search friction
JEL Classifications: G10, G12, G21

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1 Introduction

The U.S. corporate bond market is an important source of financing for firms,[1] and it has a decentralized over-the-counter (OTC) structure.[2] Empirical studies starting from Collin-Dufresn, Goldstein, and Martin (2001) document that there is a systemic non-default component in all corporate bonds' yield spread changes over time. This component can not be explained by bond, firm or macroeconomic-specific fundamentals. Later studies show that this systemic component is closely related to a market-wide liquidity factor. Motivated by the theoretical rationalization on that OTC market frictions drive the liquidity-related part of transaction price in decentralized markets, Friewald and Nagler (2019) empirically show that OTC market frictions, namely systemic inventory, search and bargaining frictions, jointly explain a large proportion of the systemic component. However, to our best knowledge, there have been very few papers focusing on bonds' heterogeneous yield spread loading on those liquidity-related frictions, and studying which market-structure factors can explain this cross-sectional heterogeneity.

In this paper, we focus on explaining bonds' heterogeneous yield spread loading on systemic search friction.[3] Since systemic search friction is one type of market-wide liquidity factor, we also call this yield spread loading as "liquidity risk from search friction" or "beta on search friction". Based on results in relevant papers, it is important to study the role of systemic search friction in driving corporate bonds' transaction price. For example, Feldhütter (2012) predicts that the premium due to search cost explains more than 10% of unexplained level of credit spread, through estimating a structural model; Friewald and Nagler (2019) find that systemic search friction explains 6.3% of the unexplained systemic

[1]Based on statistics from Securities Industry and Financial Markets Association (SIFMA), the new issuances of U.S. corporate bonds constitutes on average 65% of new capital issuance through 2009-2019, and the market total outstanding amount reached $9.6 trillion by 2019.

[2]Although U.S. corporate bond market has experienced a steady growth in centralized electronic transactions in recent years, the proportion of electronic transactions still remains low. For example, O'Hara and Zhou (2021) shows that by 2017, only around 13% of notional amounts or 25% of trades happen via a main electronic venue, MarketAxess. And this platform accounts for 85% of market share among all electronic trading platforms for U.S. corporate bonds. Therefore the majority of corporate bond transactions still remain voice-based and happen in a decentralized structure.

[3]In this paper, we define systemic search friction as the average level of search friction across all bonds at each time point, similar as Friewald and Nagler (2019). Therefore, at each time point, the value of systemic search friction is same for all bonds and is also positively correlated with the value of each bond's own search friction.

component of yield spread changes. It is also important to study bond-level liquidity risk from search friction, because, based on our sample, there are 61% of bond-and-year[4] having yield spread increase with search friction (liquidity risk takes negative value[5]), and the other pairs of bond-and-year have yield spread decrease with search friction (liquidity risk takes positive value). Within the negative-liquidity-risk bonds, a higher *absolute* value of liquidity risk implies that the bond's yield spread increases by higher amount for per unit increase in systemic search friction. A one standard deviation increase in the *absolute* value is on average associated with 8.4 bps increase in the level of yield spread, across all subperiods. This compensation for liquidity risk from search friction is highest in the Post-crisis period[6] when it is equal to 34 bps, and the second highest is equal to 24 bps, which happens in the Crisis period; correspondingly, within the positive-liquidity-risk bonds, since these bonds can be used to hedge search friction risk, a higher *absolute* value of liquidity risk implies that the bond's yield spread decreases more for per unit increase in search friction. Therefore, explaining the cross-sectional heterogeneity in bond's liquidity risk from search friction will help explain the cross-section of yield spread levels.

To explain the cross-sectional heterogeneity, we propose a measure of bond misallocation which is defined as the covariance of bond-traders' two idiosyncratic states: private-valuation type for holding a bond and inventory position on the bond. Similar as Duffie, Gârleanu, and Pedersen (2005), private valuation is same as traders' idiosyncratic preference for a bond, which is sourced from each trader's balance sheet cost, hedging liquidity need, dash for cash, etc. A trader, whose private-valuation type for a bond is lower (higher) than other traders, more prefers holding cash (the bond) to holding the bond (cash). Therefore, the covariance above describes, for each bond-and-period, how the bond's positions are allocated among traders with different preferences. In a frictionless market with no search friction, all positions are held by traders with the highest private-valuation type(s). However, in OTC

[4]For each pair of bond j and year y, we collect all transactions of bond j happening within a three-year time window $[y-2, y]$ and use the observations to estimate bond j's liquidity risk from search friction within time window $[y-2, y]$.

[5]Later we will show that liquidity risk takes the same sign with the derivative of bond price with respect to search friction. Since price moves in opposite direction with yield spread, when a bond's liquidity risk takes a negative (positive) value, the bond's price decreases (increases) with search friction and its yield spread increases (decreases) with search friction.

[6]We follow Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) to divide the whole sample period into five subperiods: Pre-crisis (Jan 2006-Jun 2007), Crisis (Jul 2007-Apr 2009), Post-crisis (May 2009-Jun 2010), Regulation (Jul 2010-Mar 2014), Volcker (post April 1, 2014).

markets with search frictions, there always exist some positions being held by traders who should have *not* held the positions in a counterfactual frictionless market. Relative to the frictionless scenario, those positions in OTC markets are regarded as "misallocated", and our proposed measure is able to capture the amount of such misallocated positions. Under a lower (higher) value of this covariance, there are more (fewer) traders with lower private valuations holding the bond in their inventories, which indicates a higher (lower) level of bond misallocation.

Theoretically and empirically, we show that, level of bond misallocation is cross-sectionally correlated with bonds' heterogeneous liquidity risk from search friction. This correlation is specific to assets which are traded in decentralized market, because it is sourced from traders' costly searching activity. Our analysis contains the following steps:

First, we use an enhanced version of U.S. Corporate Bond TRACE data, which is provided by FINRA, to test whether bonds have significantly heterogeneous liquidity risk from search friction. The data allows us to identify the two counterparties for each realized transaction.[7] We use the length of intermediation chain as a metric of the over-the-counter search friction, which is rationalized in Shen, Wei, and Yan (2018), Neklyudov and Sambalaibat (2018) and Hugonnier, Lester, and Weill (2020). The length of intermediation chain is defined as the number of traders (who are the identified FINRA's member firms in data) providing intermediation service for a transaction of some volume of a bond between an initial seller and a final buyer.[8] The average length of intermediation chain across all bonds is expected to decrease with systemic search friction, because (i) lower search friction encourages more traders to buy and sell same volume of bond at different prices to gain intermediation profit, thus raising the intensity of reallocations between traders, or (ii) a larger intermediation sector, which includes traders trading on both buy and sell sides, more facilitates transactions, thus reducing search friction.[9] Then we estimate bond-specific liquidity risk from search

[7]In this data, trading counterparties who are FINRA's member firms are mostly broker-dealers, and in most related papers, such counterparties are identified as "dealers"; trading counterparties who are not FINRA's member firms are outside institutional or retail investors and they are identified as "customers".

[8]In data, we identify initial sellers (final buyers) as the investors who are not FINRA's member firms and sell bonds to (buy bonds from) FINRA's member firms.

[9]As in Friewald and Nagler (2018), we do not discuss the endogeneity issue between length of intermediation chain and level of search friction. We only focus on the negative correlation between the two moments. Since length of intermediation chain is a more explicit and computable measure, we therefore use it as a measure of search friction which is more difficult to observe.

friction in a multi-factor model. In the cross-section of bonds, the standard deviation of the estimated liquidity risk is more than three times of its mean level.

Second, we build a search-and-match model to study which market-structure factors can explain the heterogeneity in bonds' liquidity risk from search friction. The model predicts that if a bond is more misallocated among its traders, the bond's price will be less sensitive to change in systemic search friction, regardless whether the bond's price moves negatively or positively with systemic search friction. This negative correlation between bond misallocation and *absolute* value of yield spread loading on search friction can be attributed to how much the marginal gain from per transaction (MG_T) contributes to the marginal gain from searching (MG_S). In OTC markets with no central exchange, traders optimally adjust their idiosyncratic searching activities to locate potential trading counterparties, so that in equilibrium, the marginal gain from searching (MG_S) equals its marginal cost (MC_S). Because marginal cost (MC_S) is positively correlated with systemic search friction, when systemic search friction increases, both the marginal cost and marginal gain increase. The marginal gain from searching (MG_S) can further be decomposed into two parts: the likelihood of trade happening (LT) conditional on searching and the marginal gain from per transaction (MG_T) conditional on trade happening, i.e., $MC_S = MG_S = LT \times MG_T$. The likelihood of trade happening (LT) is positively correlated with how bond positions are misallocated among all traders. The marginal gain from per transaction (MG_T) depends on the gap between a trader's private valuation and her realized transaction price. When a bond's positions are more misallocated such that more lower-private-valuation traders hold the bond, for all traders as a whole, they have a stronger incentive to invest in higher searching intensity to reallocate the bond positions among themselves. In this case, it will be faster for every trader to meet a counterparty with positive trading surplus, i.e., the likelihood of trade happening (LT) is high. Therefore, at each given level of systemic search friction, the contribution of the likelihood of trade happening (LT) to the marginal gain from searching (MG_S) is also high. By equality $MC_S = MG_S = LT \times MG_T$, for per unit change in search friction (i.e., marginal cost of searching activity (MC_S)), the marginal gain from per transaction (MG_T), which is directly associated with realized transaction price, will change correspondingly by lower magnitude.

A more intuitive example is, assume there is a trader who holds some position of a bond and has a high "dash for cash". The trader will obtain a very low utility unless she is able to offload her holding position to other traders before some specific time. For this trader to succeed in selling the bond to others, it is crucial that the other traders, at least those with positive trading surplus with her, choose high search intensity, which raises the probability that the trader meets a trading counterparty in a short time. The other traders will choose high search intensity either when (i) the bond is more misallocated within the market so that the likelihood of trade happening (LT) conditional on searching is high enough, or when (ii) the marginal gain from per transaction (MG_T) is high enough conditional on trade happening. Suppose the level of bond misallocation is low such that the first term LT is low, when systemic search friction increases (i.e. $LT \times MG_T$ increases), the only way by which the trader can motivate other traders to choose high search intensity and maintains the easiness of trade is to accept a much lower sale price than before, i.e., by raising the second term MG_T. In this case, one unit increase in systemic search friction leads to a large amount of decrease in bond price (increase in yield spread).

As for whether transaction price increases or decreases with systemic search friction, this depends on whether there exist a nontrivial amount of traders who search to sell (buy) the bond, and also depends on how strong incentives the traders have to offload their bond positions to others (buy the bond into their inventories). For example, a bond will have its average transaction price (yield spread) decrease (increase) with systemic search friction, if there are a nontrivial group of traders paying very high cost for holding the bond. These traders will be more willing to accept a lower transaction price to offload the bond as soon as possible, when it becomes more difficult to meet a counterparty with positive trading surplus. It is the opposite situation for a bond with low holding costs for all traders. In this case, the bond's average transaction price (yield spread) may increase (decrease) with search friction, because the bond is popular to all traders, due to its low holding cost. When it is more difficult to locate the bond, buying-traders are more willing to offer a higher transaction price to buy the bond.

Finally, we estimate each bond's misallocation among traders[10] based on an extended setting of the model, and show its significant correlation with bond's heterogeneous liquidity risk from search friction in the cross section. Within each bond-and-month, for all traders

[10]In data, the traders are identified as FINRA's member firms. Because we can observe each member firm's virtual ID, we can track each firm's cumulative buying and selling amounts over time, and thus estimating their holding positions on each bond and at each time. Therefore within each period, we can characterize how a bond's positions are allocated among all the firms.

who trade this bond, we separately estimate these traders' two idiosyncratic states, namely the traders' private valuation for holding the bond and their inventory position on the bond. For private valuation, we propose an estimate based on traders' completed transactions. It is calculated as the average of each trader's maximum (orthogonalized) buying price and minimum (orthogonalized) selling price, and both prices are orthogonalized against changes in bond's fundamental value; for inventory position, we follow the approach in Hansch, Naik, and Viswanathan (1998) to estimate traders' standardized inventory position using their consecutive buying and selling amounts. Then within each bond, we calculate the misallocation as a cross-trader covariance of the above two estimated series. We construct a panel data which contains all bonds' yearly series of estimated misallocation and liquidity risk from search friction. We verify the model prediction that across all bonds, a higher level of misallocation is associated with a lower *absolute* value of liquidity risk from search friction, for both negative- and positive-liquidity-risk bonds. This finding also supports that, in a decentralized financial market, the joint distribution of market participants' idiosyncratic states is correlated with the intensity at which market-wide search friction drives the asset price. We also empirically verify another two model predictions, which help us understand the channel through which bond misallocation cross-sectionally determines liquidity risk from search friction.

1.1 Related literature

This paper contributes to the empirical literature initiated by Collin-Dufresn, Goldstein, and Martin (2001) that uncovers fundamental factors to explain U.S. corporate bonds' yield spread variations over time. In this literature, Collin-Dufresn, Goldstein, and Martin (2001) establish that there is an unexplained single common factor in corporate bonds' yield spreads after controlling for commonly used explanatory variables; Longstaff, Mithal, and Neis (2005) measure the size of the default and non-default components in corporate yield spreads, and show that the non-default component is related to bond-specific as well as macroeconomic measures of liquidity. Later papers add in other common risk factors to improve the explanation, see Bao, Pan, and Wang (2011), Crotty (2013), Friewald and Nagler (2019), He, Khorrami, and Song (2019), and etc. In particular, Friewald and Nagler (2019) attribute the unexplained part of the non-default component to over-the-counter (OTC) market frictions.

Some papers target on explaining bond returns instead of change in yield spread, including De Jong and Driessen (2012), Friewald and Nagler (2016), Bongaerts, De Jong, and Driessen (2017), Bai, Bali, and Wen (2019), Bali, Subrahmanyam, and Wen (2021), Goldberg and Nozawa (2021), and etc. Our paper is more related to Goldberg and Nozawa (2021), who analyzes how dealers' liquidity supply through inventory positions drives bonds' expected return. Our paper differs in that, we focus on the *distribution* of inventory positions among different traders, which determines the cross-sectional heterogeneity in bonds' yield spread loading on the common systemic search friction.

This paper is also related to a series of papers which focus on the relationship between pricing of fixed income securities and liquidity provision in over-the-counter market. Among these papers, Bessembinder, Spatt, and Venkataraman (2019) gives a complete review on the decentralized microstructure of fixed income securities; some papers study the effect of intermediation and trading relationship on asset price, including Di Maggio, Kerman, and Song (2017), Goldstein and Hotchkiss (2018), Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018); some papers focus on the effect of market fundamentals, e.g. search friction and trading venues (voice-based or electronic), including Feldhütter (2012) and Hendershott and Madhavan (2015). The model estimated in this paper also relates to a multi-factor and liquidity-based asset pricing framework, which includes works by Fama and French (1993), Carhart (1997), Pastor and Stambaugh (2003), and Acharya and Pedersen (2005).

The theoretical part of this paper is related to a literature initiated by Duffie, Gârleanu, and Pedersen (2005) who use a search-and-match model to study asset price and liquidity in OTC markets. Our model maintains the assumption on a random search environment, which is similar to one strand of the literature including Duffie, Gârleanu, and Pedersen (2007), Vayanos and Wang (2007), Vayanos and Weill (2008), Weill (2008), Praz (2014), Afonso and Lagos (2015), Atkeson, Eisfeldt, and Weill (2015), Trejos and Wright (2016), and Hugonnier, Lester, and Weill (2020), etc. For the purpose of our analysis, our model only contains one short period, during which each trader either trade or not trade. It is also essential that we allow traders to adjust their idiosyncratic searching activity based on their idiosyncratic states and market fundamentals. This setting is similar to another strand of the literature, including Shen, Wei, and Yan (2018), Neklyudov (2012), Milbradt (2016), Farboodi, Jarosch, and Shimer (2017) and Üslü (2019). Most papers in the theoretical literature focus on how searching activity determines the transaction volume and price *within every pair of two* *counterparties*. This paper instead focuses on how search friction drives asset's average transaction price across all traders.

The rest of the paper is organized as follows: section 2 estimates bond-level liquidity risk from search friction, and characterizes the cross-sectional heterogeneity in this type of liquidity risk among all bonds. Section 3 proposes a measure of bond misallocation among traders based on a simple search-and-match model, then verifies how bond misallocation cross-sectionally explains liquidity risk from search friction. In particular, section 3.1 builds and solves the search-and-match model; section 3.2 estimates bond-level misallocation in the data; section 3.3 validates the relationship between misallocation and liquidity risk; section 3.4 further validates another two model predictions, which helps us understand the channel through which bond misallocation determines liquidity risk. Section 4 concludes.

2 Liquidity risk from search friction

We estimate bond-level liquidity risk from search friction by fitting a multi-factor model for bond yield spread, similar to Friewald and Nagler (2019). We use the volume-weighted average length of intermediation chain as a measure of systemic search friction[11], and we regard bond's yield spread[12] loading on the average chain length as bond's liquidity risk from search friction. We will show that, in cross section of bonds, the value of this liquidity risk varies across bonds.

Data We use an enhanced version of U.S. Corporate Bond TRACE Data which includes all realized corporate bond transactions and is provided by the Financial Industry Regulatory Authority (FINRA). In this data, transactions happen either between a FINRA's member firm[13] and an outside non-member firm, or between two FINRA's member firms.

[11] The weighted average length is also the average number of traders participating in the intermediation process. Details about how the intermediation chains are constructed in data are discussed in Appendix B.

[12] Yield spread is defined as the difference between corporate bond yield and the treasury yield whose term equals the corporate bond duration. Similar to Crotty (2013), Friewald and Nagler (2019), etc, we calculate treasury yields of different terms through linearly interpolating between points on the treasury curve.

[13] FINRA's member firms are mostly broker-dealers, exchanges and crowd-funding portals, which are regulated by FINRA. Member firms are required to submit reports to FINRA after they complete transactions on corporate bonds. The reports include detailed information on realized transactions, including bond ID, counterparty ID, price, volume, execution time, etc, and each report must be submitted within 15 minutes since the corresponding transaction happens.

One advantage of the dataset is, we can observe the virtual IDs[14] of FINRA's member firms in all completed transactions. Therefore in data, we identify all FINRA's member firms as "traders". This data allows us to track, for each volume of bond sold from a non-member firm to a member firm, how this volume of bond is transacted between traders (i.e. member firms) before finally sold to another non-member firm. Therefore, we are able to identify intermediation chains for each bond. It also allows us to characterize how a bond's positions are allocated among all traders. Then we are able to build the measure of bond misallocation, which is an important cross-sectional determinant of bond's liquidity risk from search friction.

We filtered the data following a similar procedure in Dick-Nielsen (2014), and additionally we recover the *real* trading counterparties in some specific types of transactions.[15] We merge the filtered data with the Mergent Fixed Income Securities Database (FISD) and Wharton Research Data Services (WRDS) Bonds Return Database to incorporate bond fundamental characteristics and credit ratings. Following the academic literature using the same dataset, we further exclude some specific types of bonds and transactions: (i) we exclude bonds with option-like characteristics, including variable coupon, convertiable, exchangable, puttable, etc, and also asset-backed securities and private placed instruments; (ii) to estimate bonds' yield spread loading on search friction, we exclude inactively traded bonds, which were traded in fewer than 25 months throught the whole sample period; (iii) we exclude all the transactions which occurred within three months after bonds' offering dates, to only consider off-the-run bond transactions.

We construct a monthly panel dataset containing both trader-level and bond-level variables[16]. The final sample ranges from Jan 2005 to Sep 2015, and contains 10760 bonds traded by 3050 traders. The total outstanding amount of all the bonds is $5.37 trillion. The average bond rating is BBB by the S&P rating categories. Among these bonds, 84% are

[14] The virtual IDs are assigned by FINRA to its member firms. Non-member firms are not assigned with such virtual IDs, so that they can not be identified in the data.

[15] For details of recovering *real* trading counterparties, see Appendix **??**.

[16] The raw data is high-frequency which records the time of each transaction accurately to the second. In the literature also using TRACE data, it is a common practice to process the data to monthly frequency because corporate bond market is relatively illiquid compared with stock markets, see Bao, Pan, and Wang (2011), Crotty (2013), Friewald and Nagler (2016), and Friewald and Nagler (2019), etc. In particular, An (2019) documents that dealers' (which are "traders" in our paper) average inventory duration in the U.S. corporate bond market is around three weeks by using the same dataset, which implies that the average frequency at which dealers adjust their inventories is around one month.

investment grade and the remaining ones are high-yield or non-rated.[17] Bonds on average have a time-to-maturity of 7.6 years. There are in all 58 million transactions with total par amount as $27.8 trillion. The average trade size is $482.4 (1,000) with a standard deviation as $4.5 (1,000). For a more complete summary statistics of our sample, see Table 7.

2.1 Measure search friction

We use length of intermediation chain as a measure of search friction in corporate bond markets, following relevant papers on empirical characterization and theory of intermediation chains. Intermediation chains were firstly constructed in Li and Schürhoff (2014) and Hollifield, Neklyudov, and Spatt (2017) to track how municipal bonds and securitization instruments are reallocated from a customer-seller to a customer-buyer through a group of dealers. The length of an intermediation chain is defined as the number of dealers, through which the asset changed hands during the reallocation process. By Hugonnier, Lester, and Weill (2020), the expected length of intermediation chain decreases with the market-wide search friction. Specifically, in a more frictional market, it is more difficult for dealers to reallocate asset with each other, so that there will be fewer dealers participating in each reallocation of the asset between customers. Then the average length of intermediation chain will be shorter.[18]

In this paper, intermediation chains are constructed and interpreted in a similar way, except now the "customer-seller" and "customer-buyer" are the outside non-member firms, and "dealers on intermediation chain" are now FINRA's member firms, i.e., the "traders" we defined above.[19]

[17]By the S&P rating categories, investment grade are S&P BBB or higher; and high-yield(junk) are below or equal to S&P BBB-.

[18]Note that length of intermediation chain is only related to number of dealers participating in each reallocation process, but not related to the physical time elapsed between when a customer sells to a dealer and when another customer buys the same volume of bond from a same/different dealer. As market-wide search friction increases, although intermediation chains will on average be shorter, it does not necessarily mean the reallocations of assets between customers also take shorter time.

[19]The reason we do not use the names of "dealers" and "customers" as in other relevant papers is, we emphasize the role of the allcation of bond positions among market participants in determining the intensity, at which systemic search friction drives bond price. Therefore, we only focus on the market participants whose bond positions can be identified and treat them equally as "traders" in our model. This helps simplify the model and more clearly show the channel through which allocation of bond positions (i.e. bond misallocation) works, without generating unnessary confusion by differentiating between dealers and customers.

To measure the systemic search friction at monthly frequency, we calculate the average length of intermediation chains across all bonds for each month, using volumes of reallocation as weights[20]. To be consistent with the model assumption in Hugonnier, Lester, and Weill (2020) that traders on an intermediation chain truely commit their capital by taking bond positions in their inventories, we only use principal trades to construct intermediation chains, by excluding pre-arranged transactions[21]. Figure 1 shows the moving average (MA) of the cross-bond average chain length over time.[22] The chain length is relatively higher before the great financial crisis, which implies lower systemic search friction in all corporate bond markets. Then it decreases during the crisis period[23] when secondary bond market liquidity significantly deteriorates. Although the average chain length recovers in the post-crisis period, after Dodd-Frank act was signed into law in July, 2010, it further decreases by nearly 8% till the third quarter of 2015. This is consistent with the effects of Dodd-Frank act on restricting broker-dealers' market-making through restricting proprietary trading practices.[24]

To further verify that the average chain length is negatively correlated with the level of search friction: (i) we plot the proportion of pre-arranged transactions among all transactions in each month. This ratio tends to be higher when market is more frictional so that traders are less willing to commit their capital to liquidity provision[25], but more willing to pre-arrange trades between buyers and sellers. In Figure 1, the ratio of pre-arranged trades is

[20]In our data, transactions happen at the trader level, instead of the firm level, which means one firm may have several trading desks located in different subsidiaries. However, since we are only able to observe traders' virtual IDs assigned by FINRA, we cannot identify those transactions that happened between subsidiaries of the same firm. If the proportion of such "within-parent-firm" transaction is high enough, this may make the average length of intermediation chains overvalued.

[21]The way by which we identify pre-arranged transactions in data is shown in Appendix A.

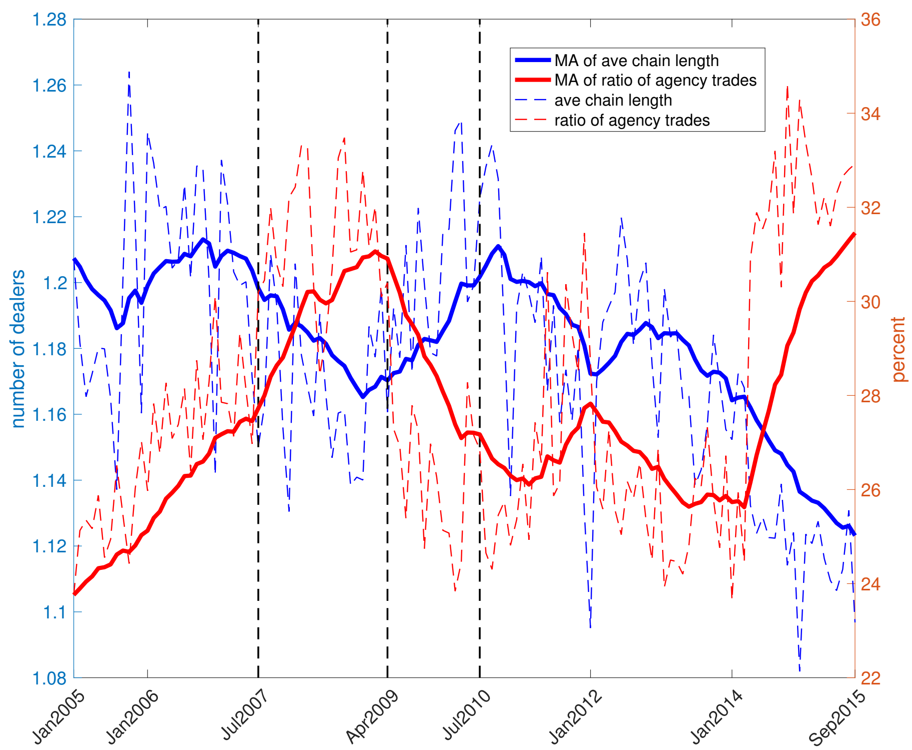
[22]For the evolutions of different compositions of the cross-bond average chain length, including volume of reallocation on each chain, different quantiles of chain length across bonds, etc, see Figure 10 and Figure 11.

[23]Similar to Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018), we divide the whole sample period into five subperiods: Pre-crisis (Jan 2006-Jun 2007), Crisis (Jul 2007-Apr 2009), Post-crisis (May 2009-Jun 2010), Regulation (Jul 2010-Mar 2014), Volcker (post April 1, 2014).

[24]This decrease in chain length was not caused by the growth in electronic trading in corporate bond market. O'Hara and Zhou (2021) shows that by 2017, only around 13% of notional amounts or 25% of trades happen via a main electronic venue, MarketAxess. And this platform accounts for 85% of market share among all electronic trading platforms for U.S. corporate bonds by Greenwich Associates 2018 Corporate Bond Trading report. Therefore, the number of proportion in notional amounts in 2015Q3 is expected to be lower than 13%, which has little effect on the calculated volume-weighted length of intermediation chain.

[25]See Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018).

Figure 1. Evolution of Search Friction (Jan 2005 - Sep 2015)



negatively correlated with the average length of intermediation chain. (ii) for each length of identified intermediation chains in data, we calculate the average length of time it takes for *each transaction on a chain* to happen. See Figure 8. The average length of time for each transaction to happen is negatively correlated with the length of intermediation chain. This also verifies that intermediation chain length is negatively correlated with intensity of trading difficulty, i.e. search friction.

2.2 Bond liquidity risk from search friction

We estimate bond-specific yield-spread loading on systemic search friction, denoted as $\beta^j_{SysSearch}$, using monthly panel data, and we use this factor loading to measure bond-specific liquidity risk from search friction. We calculate yield spread as the gap between bond yield and the same-maturity treasury yield which is obtained by interpolating yield curve. Then we estimate the target factor loading $\beta^j_{SysSearch}$ in a multi-factor model with controls on other OTC market frictions (Friewald and Nagler (2019)), market risk factors (e.g. equity pricing factors, market volatility, etc, in Fama and French (1993), Carhart (1997), Crotty (2013)), and bond-specific fundamentals. The factor loading $\beta^j_{SysSearch}$ measures how sensitively the non-default component of yield spread of bond j responds to change in systemic search friction. Systemic search friction is measured by the volume-weighted average length of intermediation chains across all bonds, and its monthly change is denoted as $\triangle SystemChainLength_t$.[26] The model is as follows:

$$\begin{aligned}\triangle(YieldSpread)_{j,t} = \beta^j_{SysSearch}\triangle SystemChainLength_t + \beta^j_{SysNetConcen}\triangle SysNetConcen_t \\ +\beta^j_{MKT}R_{MKT,t} + \beta^j_{SMB}R_{SMB,t} + \beta^j_{HML}R_{HML,t} + \beta^j_{UMD}R_{UMD,t} \\ +\boldsymbol{\gamma}^j_1\triangle I_t + \boldsymbol{\gamma}^j_2\triangle B_t + \boldsymbol{\gamma}^j_3\triangle X^{(j)}_t + \epsilon_{j,t}\end{aligned} \tag{1}$$

and our main focus is to discuss how market-structure factors cross-sectionally determine the value of $\beta^j_{SysSearch}$.

The other controls in the multi-factor model are as follows: (i) change in trader-network concentration $\triangle SysNetConcen_t$, which is the summation of all traders' average degree centralities[27] in month t; (ii) returns on factor-portfolios $R_{MKT,t}$, $R_{SMB,t}$, $R_{HML,t}$ and $R_{UMD,t}$, namely market portfolio (S&P 500 portfolio), small-minus-big(SMB) portfolio, high-minus-low(HML) portfolio and up-minus-down(UMD) momentum-factor portfolio; (iii) change in inventory-related frictions $\triangle I_t = (\triangle inv_{t-1}; \triangle amtout_t; \triangle prearrange_t)$, in which $\triangle inv_{t-1}$ is the one-month-lagged change in all traders' inventories in all bonds, $\triangle amtout_t$ is the change in all bonds' total outstanding amount, $\triangle prearrange_t$ is the change in pre-arranged ratio of all transactions; (iv) change in bargaining frictions $\triangle B_t = (\triangle blocktrade_t;$

[26] We use the average chain length across all bonds to make the *level* of search friction same to all the bonds. Because our focus is to characterize and explain the cross-sectional heterogeneity in bond's yield-spread loading on the *common* search friction, we use the cross-bond average chain length to avoid the case that bond's liquidity risk from search friction is correlated with current level of search friction.

[27] Degree centrality is another measure of vertices' centralities in a network. Unlike eigenvector centrality, degree centrality only takes into account all direct links directed from or to each vertice. For a network with n vertices, the theoretical maximum value of the summation of all vertices' degree centralities is $n(n-1)$. Therefore, the summation of all traders' degree centralities in the trader-network is a better measure of the concentration of the network. The closer the summation is to $n(n-1)$, where n is the number of traders, the less concentrated the trader-network is.

$\triangle HHItrader_t$), in which $\triangle blocktrade_t$ is the change in the proportion of block trades, and $\triangle HHItrader_t$ is the change in the average value of all bonds' HHI indices[28]; (e) all the other bond-level and market-level controls $\triangle X_t = (\triangle(YieldSpread)_{j,t-1}, \triangle RF_t; (\triangle RF_t)^2; \triangle SLOPE_t; \triangle turnover_t^j; Rating_t^j; TTM_t^j)$, in which $\triangle(YieldSpread)_{j,t-1}$ is the lag term of change in yield spread, $\triangle RF_t$ ($(\triangle RF_t)^2$) is the (squared) change in 10-year treasury rate, $\triangle SLOPE_t$ is the change in slope of yield curve, $\triangle turnover_t^j$ is the change in bond j's current-month turnover rate, $Rating_t^j$ is bond j's credit rating in month t and TTM_t^j is bond j's time to maturity in month t.

The *average* value of $\beta^j_{SysSearch}$ across all bonds is significantly negative, as shown in Table 1. This is consistent with results in Friewald and Nagler (2019) who only reports the cross-bond average value of $\beta^j_{SysSearch}$. This implies that, on average across all bonds, a one standard deviation increase in intermediation chain length (decrease in systemic search friction) is associated with *at least* 8.57 bps decrease in yield spread.[29] For full regression results, see Table 18 in Internet Appendix.

2.2.1 Cross-sectional heterogeneity in liquidity risk from search friction

However, the value of $\beta^j_{SysSearch}$ significantly varies across bonds. To better characterize the full distribution of $\beta^j_{SysSearch}$ and its potential cross-sectional determinants, we separately estimate group- and bond-level $\beta^j_{SysSearch}$ using our sample data.

For group-level $\beta^j_{SysSearch}$, bonds are classified into different groups based on contract terms and issuer characteristics[30]. The estimation result shows that for most groups, group-level $\beta^j_{SysSearch}$ is negative but with different *absolute* values. A group with a higher *absolute* value of $\beta^j_{SysSearch}$ will have its bonds' price change more sensitively with systemic search

[28] Block trades are defined as trades with volume larger than $1,000,000. Each bond's HHI index is calculated by using all traders' market shares in that bond. Both variables are proxy for systemic bargaining frictions in the U.S. corporate bond market: the higher the ratio of block trades is, the more bargaining power the outside non-member firms have, and the higher the average value of all bonds' HHI indices is, the more concentrated are bonds' transactions to a subset of traders, therefore, the lower bargaining power the outside non-member firms have.

[29] We use the estimate of coefficient of $\triangle SystemChainLength_t$ in specification (3) of Table 1, which controls all OTC market frictions and bond-specific characteristics. This specification generates the average bond-specific liquidity risk with the lowest *absolute* value. In the distribution of intermediation chain length, 95% of observations fall within around four standard deviations of the mean.

[30] For contract terms, we consider time-to-maturity, outstanding amount, credit rating; for issuer characteristics, we consider issuers' leverage ratio, book-to-market ratio, profitability(ROA), and industry sector.

Table 1. Average of bond-specific liquidity risk

	Dependent variable:		
	$\triangle(YieldSpread)_{j,t}$ (%)		
	(1)	(2)	(3)
$\triangle SystemChainLength_t$	-2.32***	-1.67***	-1.55***
	(-32.80)	(-21.38)	(-21.38)
$\triangle SysNetConcen_t$ (thousand)	-9.83e-03***	-4.77e-03***	-4.43e-03***
	(-48.16)	(-22.30)	(-20.10)
Mean Adjusted R^2	0.18	0.35	0.37
# of Bonds	11,176	11,176	9,595
Observations	515,514	515,514	479,146
inventory and bargaining frictions	YES	YES	YES
market aggregates and FFC 4 factors	NO	YES	YES
bond liquidity and fundamentals	NO	NO	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets. We exclude bonds with total number of observations smaller or equal to 19 for column (1)-(2) and smaller or equal to 25 for column (3). The reported estimated coefficients are average values taken across all bonds. The corresponding t-statistics are calculated by dividing each reported (average) coefficient value by the standard deviation of the estimates and scaling by the square root of the number of bonds. Details are in section **IA.2** of Internet Appendix.

friction. Also, group-level $\beta^j_{SysSearch}$ significantly but *non-linearly* depends on most of the contract terms and issuer characteristics. This motivates us to find a cross-sectional determinant of $\beta^j_{SysSearch}$ which is potentially related to market structure and distribution of traders' characteristics, and determines bonds' liquidity risk from search friction in a more linear way. The cross section of group-level $\beta^j_{SysSearch}$ are shown in Figure 5 and Figure 6. For related regression results, see Table 13, Table 14, and Table 15 in Internet Appendix.

The estimation result of bond-specific $\beta^j_{SysSearch}$ implies that, individual bonds have their yield spreads respond, either negatively or positively, to increase in systemic search friction. There exist quite a portion of bonds having their prices increase when search friction increases (i.e., positive $\beta^j_{SysSearch}$). Similar to group-level estimates, the *absolute* value of bond-specific $\beta^j_{SysSearch}$ also significantly varies across bonds. The cross section of bond-specific $\beta^j_{SysSearch}$

are shown in Figure 7.

2.2.2 Liquidity risk and yield spread level

In this section, we show that it is important to characterize and explain the cross-sectional heterogeneity in corporate bond's liquidity risk from search friction, because this type of liquidity risk is significantly compensated by yield spread and thus helping explain the cross-section of yield spread levels.

We run Fama-MacBeth regression for different subperiods. Specifically, we follow Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) to divide the whole sample period into five subperiods: Pre-crisis (Jan 2006-Jun 2007), Crisis (Jul 2007-Apr 2009), Post-crisis (May 2009-Jun 2010), Regulation (Jul 2010-Mar 2014), Volcker (post April 1, 2014). Within each subperiod, we firstly estimate bond-specific liquidity risk $\beta^{j,s}_{SysSearch}$, $s = 1, 2, 3, 4, 5$ in model (1). Then in the second step, we run cross-sectional regression of *level* of yield spread on estimated factor loadings from the first step and other determinants, separately for each subperiod s:

$$YieldSpread_{j,s} = \lambda^s_{SysSearch} * \beta^j_{SysSearch,s} + \lambda^s_{SysNetConcen} * \beta^j_{SysNetConcen,s} \tag{2}$$

$$+\lambda^s_{prearrange} * \gamma^j_{1,prearrange,s} + \lambda^s_{inv} * \gamma^j_{1,inv,s}$$

$$+\lambda^s_{blocktrade} * \gamma^j_{2,blocktrade,s} + \lambda^s_{HHItrader} * \gamma^j_{2,HHItrader,s} + \overline{BF}^j_s + \eta_s + \epsilon^j_s$$

where $s = 1, 2, 3, 4, 5$. $\{\beta^j_{SysSearch,s}, \beta^j_{SysNetConcen,s}, \gamma^j_{1,prearrange,s}, \gamma^j_{1,inv,s}, \gamma^j_{2,blocktrade,s}, \gamma^j_{2,HHItrader,s}\}$ are the collection of estimated factor loadings from the first step, which measures bonds' liquidity risk from search friction and other bond-specific yield-spread loadings on trader-network concentration, ratio of pre-arranged trades, traders' aggregate inventory, ratio of block trades, and the competition level of trader market. $\overline{BF}^j_s$ is a collection of bond-level determinants of spreads, including bond liquidity measured by Amihud[31], trade concentra-

[31] $Amihud^j_t$ is a liquidity measure proposed by Amihud (2002), which is calculated as the average *absolute* value of daily return divided by daily par dollar volume. Specifically, $Amihud^j_t = \frac{1}{d_{j,t}} \sum_{\tau=1}^{d_{j,t}} \frac{|r_{j,\tau}|}{Volume_{j,\tau}}$, where $d_{j,t}$ is the number of days with observed returns in month t for bond j, $r_{j,\tau}$ is the return for bond j on day τ, and $Volume_{j,t}$ is the par dollar volume traded on day τ. Within each subperiod s, we calculate volume-weighted average $Amihud^j_s$, using monthly trading volume as weights.

tion (among traders), credit rating, bond-specific search friction[32] and number of trades in segmented markets.[33]

We are specifically interested in $\lambda^s_{SysSearch}$, $s = 1, 2, 3, 4, 5$, which characterizes the cross-sectional relationship between bond liquidity risk from search friction and yield spread. We show in Table 2 that the estimate of $\lambda^s_{SysSearch}$ is uniformly negative across subperiods. This implies that, for a bond with negative liquidity risk ($\beta^j_{SysSearch,s} < 0$), the bond's yield increases with search friction, and the more sensitively the bond's yield increases with search friction, its average yield will be higher to compensate holders for the higher liquidity risk; for a bond with positive liquidity risk ($\beta^j_{SysSearch,s} > 0$), the bond's yield decreases with search friction, and the more sensitively the bond's yield decreases with search friction, its average yield will be lower because holders need to pay for the heding function of the bond. By Table 2, on average across all subperiods, a one standard deviation (13.7) increase in the *absolute* value of $\beta^j_{SysSearch,s}$ (for $\beta^j_{SysSearch,s} < 0$) is associated with a 8.4 bps higher yield spread. This compensation for liquidity risk from search friction is highest in the Post-crisis period when it is equal to 34 bps, and the second highest is equal to 24 bps, which happens in the Crisis period.[34] For full regression results, see Table 8.

Our next step is to construct a new bond-level measure, bond misallocation among traders, and use it as a cross-sectional determinant of the *absolute* value of bond's liquidity risk from search friction, after controlling bonds' contract terms and issuer characteristics.

3 Cross-sectional determinants of liquidity risk from search friction

In this section, we build a simple search-and-match model with traders trading a single bond, and based on the model, in Section 3.1.3, we formally construct the measure of bond

[32]Bond-specific search friction is measured by the average length of time interval between consecutive trades on each intermediation chain, excluding the head and tail trades. The reason we exclude the head and tail segments of intermediation chains is that these trades are more likely to be pre-arranged or more likely imply directed search of outside non-member firms instead of the random search of traders we focus on.

[33]We consider two segmented markets, one including transactions only between FINRA's member firms, the other including transactions between FINRA's member firms and outside non-member firms.

[34]Note that the increase in yield spread level depends on both the value of $\lambda^s_{SysSearch}$ and standard deviation of liquidity risk in each subperiod.

Table 2. Fama-Macbeth regression of yield spread level on factor loadings

Subperiod	*pre-crisis* ($s = 1$)	*crisis* ($s = 2$)	*post-crisis* ($s = 3$)	*regulation* ($s = 4$)	*Volcker* ($s = 5$)
Standard deviation of $\beta^{j}_{SysSearch,s}$	7.14	27.02	9.19	6.41	23.70
$\lambda^{s}_{SysSearch}$ (bps)	-1.21***	-0.89***	-3.65***	-1.13***	-0.89***
	(-3.43)	(-5.70)	(-10.46)	(-6.00)	(-5.18)
Adjusted R^2	0.48	0.51	0.62	0.45	0.42
# of Observations	2371	2468	2517	7078	2958

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets.

misallocation among traders. Excluding heterogeneity in bond-specific characteristics, this model predicts that, a bond with more (less) positions being misallocated among traders has a lower (higher) *absolute* value of yield spread loading on systemic search friction. The key model component which leads to this prediction is traders' choosing their search intensity.[35] More misallocated bond positions motivate traders to invest in higher search intensity to reallocate the bond among themselves. Then for both the traders who wish to offload their holding positions and the traders who wish to fill their balance sheet space, they can more quickly complete the transaction without making their accepted price change too much. Therefore, in a market with a higher level of bond misallocation, bond price is more rigid to change in systemic search friction. In other words, the *absolute* value of bond's liquidity risk from search friction is relatively low.

[35] Same as the literature of search-and-match model for over-the-counter markets, a trader's search intensity (or efforts spent on searching activity) determines the likelihood that she is matched with another counterparty to bargain and trade with, during the random search process.

3.1 A simplified model

3.1.1 Setting

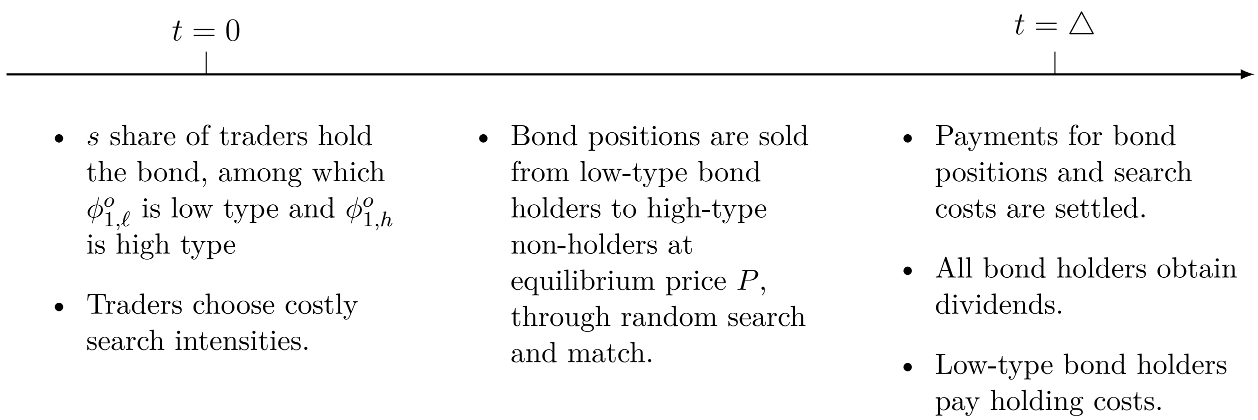
We consider a simplified one-period search-and-match model with a continuum of traders trading an indivisible bond of one-period maturity. The length of period is denoted as $\triangle$, and we assume in value $0 < \triangle << 1$. The bond only pays a dividend $d > 0$ at maturity. The measure of all traders is normalized to one, and each trader's bond position a is assumed to be either zero or one.[36] Besides bond positions, the traders are also characterized by an private valuation δ which is either low or high, and denoted by $\delta \in \{\ell, h\}$. We assume the private valuation follows a discrete uniform distribution among the traders with PDF $P(\delta = \hat{\delta}) = \Phi_{\hat{\delta}} = \frac{1}{2}$, $\hat{\delta} \in \{\ell, h\}$. Following the interpretation of private valuation in the Introduction session, we assume that, by holding the bond at maturity ($t = \triangle$), a low-type trader needs to additionally pay a holding cost $\overline{C} > 0$ which satisfies $d - \overline{C} < 0$, and a high-type trader does not need to pay such cost.[37]

The economy has two time points, $t = 0, \triangle$. Figure 2 describes the timeline. At $t = 0$, a fraction $0 < s < 1$ of traders each initially hold one unit of the bond, and we also call them "bond holders". Among these bond holders, a measure $\phi^o_{1,\ell}$ of the bond holders have a low-type private valuation, and the remaining $\phi^o_{1,h} = s - \phi^o_{1,\ell}$ of the bond holders have a high-type private valuation. There is no asymmetric information in the economy. All traders are risk neutral, and they discount payment flows at rate r. They each choose a costly search intensity based on their own private valuations, bond positions and their expectations on the distribution of demographics in the market. During the single period, traders randomly search and trade with each other in the market. Intuitively, the most essential motivation for traders to trade is: the low-type traders who hold the bond want to offload their positions to high-type traders who do not hold the bond. A trader with a higher search intensity is able to more quickly meet another trader in the market, so that there is a higher probability that the trader completes a transaction within the short period $\triangle$.

[36] We borrow the $\{0, 1\}$ assumption for bond holding from Duffie, Gârleanu, and Pedersen (2005). This assumption, together with the indivisibility of the bond, determines that the trading volume in each transaction equals one unit, and traders cannot short-sell the bond.

[37] This setting follows Duffie, Gârleanu, and Pedersen (2005). The holding cost can be interpreted as from traders' need for cash, high financing costs and hedging needs to sell, etc. In this paper, we specifically consider high-enough holding cost, which motivates low-type traders who hold the bond to "compete" selling the bond to high-type traders, otherwise, the low-type holders will obtain a negative cash flow at maturity.

Figure 2. Timeline of the Economy



We assume the following specification of matching function: the intensity at which a trader with search intensity λ contacts *and* is contacted by another trader with search intensity λ' equals $m(\lambda, \lambda') = 2\lambda\lambda'$. More formally, the length of time before a trader meets a counterparty with positive trading surplus follows an exponential distribution. The exponential parameter is jointly determined by the trader's search intensity, the counterparty's search intensity, the counterparty's population size, and market fundamentals. Details are in trader's problem (3). Bond positions are sold from low-type bond holders to matched high-type non-holders at an equilibrium price P. [38] At $t = \triangle$, payments for transactions, search costs and holding costs of low-type bond holders are settled, and dividends are also paid to all bond holders.

Trader's problem We consider each trader's maximizing her discounted net payment flow at $t = 0$, by choosing search intensity λ at a quadratic form of flow cost[39] $\kappa\lambda^2$, $\kappa > 0$. Denote trader's discounted net payment flow as $U_a(\delta)$, with $a \in \{0, 1\}$ and $\delta \in \{\ell, h\}$, which

[38]In our model with two discrete private valuations, there only exists positive trading surplus between a low-type bond holder and a high-type non-holder. We also assume that each trader at most trades for one time during the short period $\triangle$, in other words, each trader either trades once or does not trade within the short period. For simplicity, in later analysis, we focus on how search cost drives the unique transaction price P between low-type and high-type traders. This price can be interpreted as the average of multiple transaction prices which happened simultaneously between different pairs of trading counterparties in over-the-counter corporate bond markets.

[39]The total search cost paid by each trader at $t = \triangle$ equals the flow cost $\kappa\lambda^2$ multiplied by the length of the period $\triangle$.

satisfies:

$$U_a(\delta) = \max_{\lambda_a(\delta)\geq 0} \quad e^{-r\triangle} \times \Bigg\{ \underbrace{\left[1 - e^{-2\lambda_a(\delta)\lambda^*_{1-a}(\delta')\phi^o_{1-a}(\delta')\triangle}\right]}_{\text{Prob. transaction happens}} \times [\mathbb{1}(a=1) \times P \tag{3}$$

$$+\mathbb{1}(a=0) \times \left(d - \mathbb{1}(\delta=\ell)\overline{C} - P\right)]$$

$$+ \underbrace{e^{-2\lambda_a(\delta)\lambda^*_{1-a}(\delta')\phi^o_{1-a}(\delta')\triangle}}_{\text{Prob. transaction not happen}} \times \mathbb{1}(a=1) \times \left(d - \mathbb{1}(\delta=\ell)\overline{C}\right)$$

$$\underbrace{-\kappa\lambda_a(\delta)^2\triangle}_{\text{search cost}} \Bigg\}$$

where $\delta, \delta' \in \{\ell, h\}$, $\delta \neq \delta'$, and the optimal solution of search intensity $\lambda_a(\delta)$ depends on current trader's bond position and private valuation. Let $\lambda^*_{1-a}(\delta')$ denote the counterparty group's optimal search intensity and $\phi^o_{1-a}(\delta')$ denote the counterparty group's population measure. On the right side, we assume the time at which a transaction happens for the current trader (a, δ), which is denoted as $\tau_{(a,\delta)}$, follows an exponential distribution with PDF $f_{\tau_{(a,\delta)}} = \hat{\lambda}(a,\delta)e^{-\hat{\lambda}(a,\delta)\times\tau_{(a,\delta)}}$, and the rate parameter $\hat{\lambda}(a,\delta) = 2\lambda_a(\delta)\lambda_{1-a}(\delta')\phi^o_{1-a}(\delta')\triangle$. No matter whether a transaction happens or not, at the end of the period, a trader choosing non-zero search intensity always needs to pay a search cost which equals $\kappa\lambda_a(\delta)^2\triangle$. The search cost is in a convex form of the chosen search intensity $\lambda_a(\delta)$ and is proportional to the length of the period $\triangle$. Given the assumption that the period is short enough, we apply the approximation rule, $e^x \approx 1 + x$ for $x \to 0$, to solve trader's problem. In particular, we assume $\triangle$ is low enough such that $2\lambda_a(\delta)\lambda_{1-a}(\delta')\phi^o_{1-a}(\delta')\triangle < 1$ in any equilibrium.

3.1.2 Model solutions

The search cost $\kappa\lambda_a(\delta)^2\triangle$ can be interpreted as traders' opportunity cost for entering and actively trading in the market. Because traders will choose positive search intensities only when the gains from searching is *strictly* positive, in trader's problem (3), the equilibrium price P must lie between low-type bond holder's net payment flow $d - \overline{C} < 0$ and high-type bond holder's net payment flow $d > 0$. Therefore at initial time $t = 0$, only low-type bond holders and high-type non-holders choose positive search intensities. To make it more "urgent" for low-type bond holders to search and offload their positions, we additionally assume the holding cost at maturity is high enough such that $d - \overline{C} = -d < 0$.

Then we define the equilibrium in the model as follows:

Definition 3.1: *Given initial distribution of traders* $\{\phi_a^o(\delta)\}_{a\in\{0,1\},\delta\in\{\ell,h\}}$ *and parameters* $\{s, \kappa, d, \overline{C}, r, \triangle\}$*, an equilibrium contains* $\{U_a(\delta), \lambda_a^*(\delta), P\}_{a\in\{0,1\},\delta\in\{\ell,h\}}$*, such that:*
(i) $\{\lambda_a^*(\delta)\}_{a\in\{0,1\},\delta\in\{\ell,h\}}$ *solve traders' problem* (3)*, and* $\lambda_0^*(\ell) = \lambda_1^*(h) = 0$*; (ii) market clears at* $t = \triangle$*, such that the total trading volume during the short period satisfies:*

$$2\lambda_1^*(\ell)\lambda_0^*(h)\phi_1^o(\ell)\phi_0^o(h)\triangle \leq min\{\phi_1^o(\ell), \phi_0^o(h)\} \tag{4}$$

The market clear condition (4) trivally applies as both $2\lambda_1^*(\ell)\lambda_0^*(h)\phi_1^o(\ell)\triangle$ and $2\lambda_1^*(\ell)\lambda_0^*(h)\phi_0^o(h)\triangle$ are lower than one.

The low-type bond holders' and high-type non-holders' optimal search intensities, $\lambda_1^*(\ell)$ and $\lambda_0^*(h)$, are complementary to each other in the sense that a high (low) search intensity on one side motivates the other side to also choose a high (low) search intensity. Therefore, if we do not normalize the optimal search intensity or the maximized discounted net payment flow of either low-type bond holders or high-type non-holders, in equilibrium only the relative search intensities $\frac{\lambda_1^*(\ell)}{\lambda_0^*(h)}$ can be uniquelly pinned down. However, in any equilibrium by Definition 3.1, the transaction price P of our main interest can be analytically solved out as follows:

$$P = \frac{2d - \overline{C} + \sqrt{(2d - \overline{C})^2 - 4(d * (d - \overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)})}}{2} \tag{5}$$

where $d - \overline{C} = -d < 0$.

Intuitively when search friction κ increases, the equilibrium transaction price P will decrease, as low-type bond holders on average give up trading surplus (i.e. accept lower sale price) to offload their bond positions to high-type non-holders as quickly as possible. This happens because low-type bond holders find it more difficult to meet a counterparty with positive trading surplus, and if no trade happens, they need to consume a negative and also large amount of payment flow $-d < 0$ at $t = \triangle$. In other words, low-type bond holders' marginal gains from per transaction decreases with search friction because of their low outside option.

3.1.3 Bond misallocation

We further analyze how the derivative $\frac{dP}{d\kappa}$, which is the theoretical counterpart of bond's liquidity risk from search friction $\beta_{SysSearch}$, depends on the initial distribution of trader demographics $\{\phi_a^o(\delta)\}_{a\in\{0,1\},\delta\in\{\ell,h\}}$. By expression (5), the higher the value of $\phi_1^o(\ell)\phi_0^o(h)$ is, the less responsive the equilibrium price P is with respect to change in search friction κ. We interpret the lower of the initial measure of low-type bond holders and that of high-type non-holders, $min\{\phi_1^o(\ell), \phi_0^o(h)\}$, as the bond positions which are misallocated among traders.[40] By model assumptions, we also have $\phi_0^o(h) = \frac{1}{2} - \phi_1^o(h) = \frac{1}{2} - (s - \phi_1^o(\ell)) = \frac{1}{2} - s + \phi_1^o(\ell)$, which requires $\phi_1^o(\ell) > s - \frac{1}{2}$. Then under the restrictions $\phi_1^o(\ell), \phi_1^o(h) \geq 0$, the level of bond misallocation $min\{\phi_1^o(\ell), \phi_0^o(h)\}$ is monotonically increasing with that of $\phi_1^o(\ell)\phi_0^o(h)$. By (5), this further implies that in our model, bond misallocation is the only determinant of transaction price's sensitivity to change in search friction, regardless of bond fundamentals. With bond positions more misallocated among traders, bond's average transaction price is less sensitive to change in search friction.

To bring our model to data, we need a more computable measure of bond misallocation which is strictly increasing or decreasing with both $min\{\phi_1^o(\ell), \phi_0^o(h)\}$ and $\phi_1^o(\ell)\phi_0^o(h)$, and is also normalized by the whole population size. We propose such a bond-level measure as the covariance between traders' holding position and their private valuation, in the cross section of traders for each bond. Note that when calculating the covariance, we assign each trader's obtained net payment flow at maturity (i.e., dividend net of holding cost) to the value of the trader's private valuation, namely $h = d$ and $\ell = d - \overline{C}$.[41] Formally, bond misallocation is

[40] The concept of "misallocation" in this paper is related to the difference in allocations of bond positions between over-the-counter and frictionless markets. In frictionless market, the equilibrium price is $P = d$ or $P = d - \overline{C}$ depending on whether the high-type or low-type traders are on the long side, i.e., whether $s < 1/2$ or $s \geq 1/2$. When high-type traders are on the long side (i.e. $s < 1/2$), at any time, no low-type traders will hold the bond, that is, $\phi_1^o(\ell) \equiv 0$. In this case, $min\{\phi_1^o(\ell), \phi_0^o(h)\} = \phi_1^o(\ell)$, so that any non-zero positions held by low-type traders, $\phi_1^o(\ell)$, will be regarded as misallocated; when low-type traders are on the long side (i.e. $s \geq 1/2$), at any time, all high-type traders will hold the bond, that is, $\phi_0^o(h) \equiv 0$. In this case, $min\{\phi_1^o(\ell), \phi_0^o(h)\} = \phi_0^o(h)$, so that the positions whose total amount equals to the total balance sheet space of high-type non-holders, $\phi_0^o(h)$, and which are also held by low-type traders, will be regarded as misallocated.

[41] The reason we do this is, across traders, the "dividend net of holding cost" is strictly monotonic with

defined as follows:

$$Cov^o(a,\delta) = \left(\frac{s}{2} - \phi_1^o(\ell)\right)(h-\ell) = \left(\frac{s}{2} - \phi_1^o(\ell)\right)\overline{C} = Cov^o(\phi_1^o(\delta),\delta) \tag{6}$$

where $\phi_1^o(\delta)$, $\delta \in \{\ell, h\}$ can also be interpreted as the expected inventory position of a δ-type trader. This is because we normalize the measure of all traders to be equal to one, which makes the population measure coincide with the probability measure.

In summary, we have three monotonicity conditions: (i) the original measure of misallocation $min\{\phi_1^o(\ell), \phi_0^o(h)\}$ is monotonically increasing with $\phi_1^o(\ell)\phi_0^o(h)$; (ii) $\phi_1^o(\ell)\phi_0^o(h)$ is monotonically increasing with $\phi_1^o(\ell)$; and (iii) by (6), $Cov^o(a,\delta)$ is monotonically decreasing with $\phi_1^o(\ell)$. By conditions (i), (ii) and (iii), the original measure $min\{\phi_1^o(\ell), \phi_0^o(h)\}$ is monotonically decreasing with $Cov^o(a,\delta)$. Then we use $Cov^o(a,\delta)$ as a computable measure of bond misallocation. In Proposition 1, we analyze the relationship between bond misallocation $Cov^o(a,\delta)$ and liquidity risk from search friction $\frac{dP}{d\kappa}$.

3.1.4 Relationship between bond misallocation and liquidity risk from search friction

Proposition 1: *Under parameter restrictions* $\phi_1^o(\ell), \phi_0^o(h) \geq 0$, $d - \overline{C} = -d < 0$, $\overline{C}^2 > \frac{4\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)}$ *and the normalization* $U_1(\ell) = 0$, *there exists a unique equilibrium by Definition 3.1 and satisfies: (i)* $\frac{dP}{d\kappa} < 0$ *and* $\frac{d}{d\phi_1^o(\ell)}|\frac{dP}{d\kappa}| < 0$*; (ii) let* $Cov^o(a,\delta)$ *denote the covariance between traders' bond positions and private valuations at* $t = 0$*, since* $Cov^o(a,\delta) \propto -1 \times \phi_1^o(\ell)$, $a \in \{0,1\}, \delta \in \{\ell, h\}$*, we have* $\frac{d}{dCov^o(a,\delta)}|\frac{dP}{d\kappa}| > 0$*. If we relax the restriction on low-type bond holder's holding cost as* $d < \overline{C} \leq 2d$*, and put additional restrictions such that* $d \geq \frac{2\kappa}{\sqrt{\phi_1^o(\ell)\phi_0^o(h)}}$, $\kappa^2 \leq \frac{1-s}{16}$ *and* $\frac{s-\frac{1}{2}+\sqrt{(\frac{1}{2}-s)^2+32\kappa^2}}{2} \leq \phi_1^o(\ell) \leq \frac{1}{2}$,[42] *then (iii) there may exist multiple equilibria when* $\overline{C}$ *takes different values:*

traders' private valuation. Moreover, it will be easier for us to characterize, under which conditions of d and $\overline{C}$, bond's price will increase or decrease with search friction.

[42] These additional restrictions are sufficient conditions, intuitively, $\kappa^2 \leq \frac{1-s}{16}$ restricts that search cost cannot be too large, so that traders have incentive to invest in search intensity to meet and trade with each other; $d \geq \frac{2\kappa}{\sqrt{\phi_1^o(\ell)\phi_0^o(h)}}$ and $\frac{s-\frac{1}{2}+\sqrt{(\frac{1}{2}-s)^2+32\kappa^2}}{2} \leq \phi_1^o(\ell) \leq \frac{1}{2}$ restrict that dividend d and the initial mis-allocated positions $\phi_1^o(\ell)$ need to be high enough relative to search cost, thus also generating high enough trading needs for traders in the market to re-allocate mis-allocated positions among themselves.

1. *if* $d < \overline{C} \leq d + \frac{\kappa^2}{d\phi_1^o(\ell)\phi_0^o(h)}$*:* $\exists$ *two equilibria, one with* $\frac{dP}{d\kappa} < 0$ *and the other with* $\frac{dP}{d\kappa} > 0$*. For both equilibria,* $\frac{d}{dCov^o(a,\delta)}|\frac{dP}{d\kappa}| > 0$*;*

2. *if* $d + \frac{\kappa^2}{d\phi_1^o(\ell)\phi_0^o(h)} < \overline{C} \leq 2d$*: there exists a unique equilibrium where* $\frac{dP}{d\kappa} < 0$ *and* $\frac{d}{dCov^o(a,\delta)}|\frac{dP}{d\kappa}| > 0$*.*

Proof of Proposition 1 is in Appendix C.1.1 and C.1.2.

The derivative $\frac{dP}{d\kappa}$ is the theoretical counterpart of bond's liquidity risk from search friction $\beta_{SysSearch}$. To formally map $\frac{dP}{d\kappa}$ to $\beta_{SysSearch}$, we need the expressions of $\frac{dYTM}{dP}$ and $\frac{dChain}{d\kappa}$ where YTM is a bond's yield-to-maturity and $Chain$ is the average length of intermediation chain for this bond. We use an approximated formula of YTM to calculate $\frac{dYTM}{dP}$: $YTM = \frac{c+\frac{F-P}{n}}{\frac{F+P}{2}}$ where we choose time-to-maturity $n = 8$, face value $F = 1$, and coupon rate $c = 0$. Then $\frac{dYTM}{dP} = -\frac{1}{2(1+P)^2}$. We calibrate the derivative of chain length with respect to search friction as -0.0045, according to the derived formula and calibrated value in Hugonnier, Lester, and Weill (2020).[43] Then we show the relationship between $\frac{dP}{d\kappa}$ and $\beta_{SysSearch}$:

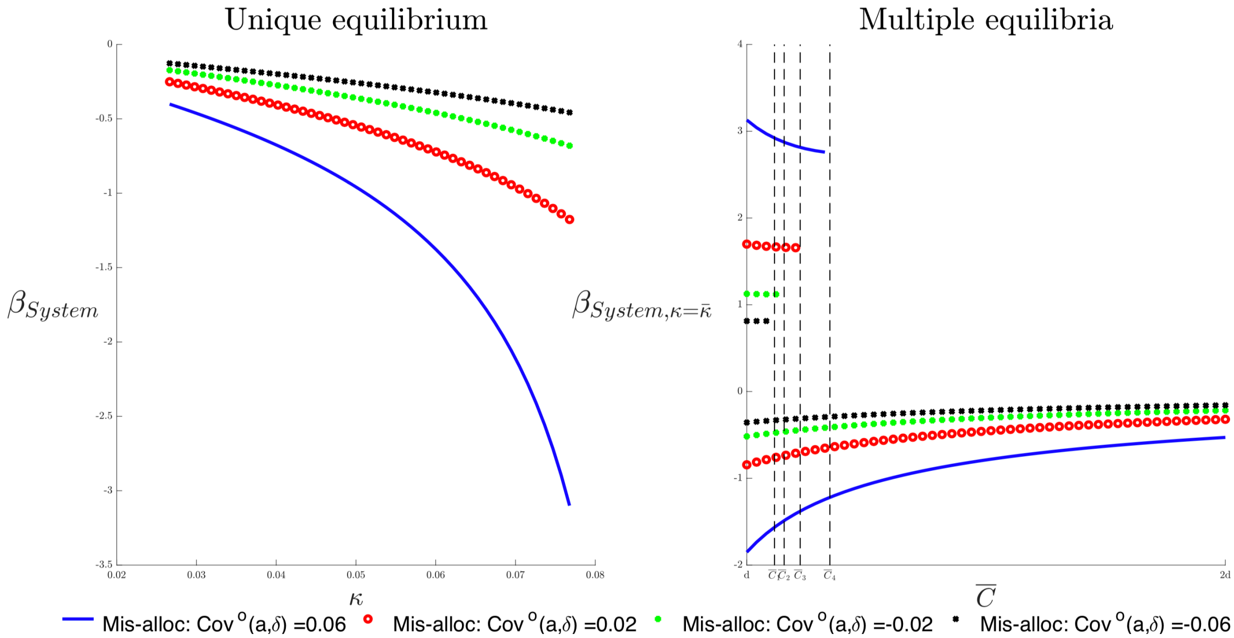
$$\beta_{SysSearch} = \frac{dYTM}{dChain} = \frac{dYTM}{dP} \times \frac{dP}{d\kappa} \times \frac{d\kappa}{dChain} = -\frac{1}{2(1+P)^2} \times \frac{dP}{d\kappa} \times \frac{1}{(-0.0045)} \quad (7)$$

Then a negative (positive) value of $\beta_{SysSearch}$ means that bond price decreases (increases) when search friction increases. By Proposition 1, a higher misallocation (i.e. lower $Cov^o(a, \delta)$) among traders makes the bond's yield spread less sensitive to change in search friction:

$$\frac{d\beta_{SysSearch}}{dCov^o(a,\delta)} < 0 \text{ if } \beta_{SysSearch} < 0; \text{ and } \frac{d\beta_{SysSearch}}{dCov^o(a,\delta)} > 0 \text{ if } \beta_{SysSearch} > 0. \quad (8)$$

[43] In Hugonnier, Lester, and Weill (2020), the expected length of intermediation chain is derived to be equal to $\left(1 + \frac{1}{\chi}\right) log(1 + \chi)$, where χ is an expression of latent parameters including market-level search intensity, distribution of demographics, and the intensity that customers' idiosyncratic private valuations change due to liquidity shocks, etc. Then χ is calibrated as solution of the equation $\left(1 + \frac{1}{\chi}\right) log(1+\chi) = 1.16$, where the right-hand side value is the volume-weighted average length of intermediation chain from our sample. The average length is taken both cross-sectionally and in time dimension. Therefore, we calibrate that $\chi = 0.356$. We further assume that the level of search friction and that of average search intensity changes one to one, by using other calibrated values in Hugonnier, Lester, and Weill (2020), we obtain that $\frac{d\chi}{d\kappa} = -\frac{d\chi}{d\lambda} \approx -0.011$, where κ denotes search friction, λ denotes average search intensity. Finally, according to the formula of average length of intermediation chain, we obtain that $\frac{dChain}{d\kappa} = \frac{dChain}{d\chi}\frac{d\chi}{d\kappa} = \frac{\chi - log(1+\chi)}{\chi^2} \times (-0.011) = -0.0045$.

Figure 3. Value of liquidity risk from search friction



Note: Level of bond misallocation decreases with the value of covariance $Cov^o(a, \delta)$.
Graph-1: fixed holding cost $\overline{C} = 2d$ and varying levels of search friction κ; Graph-2: fixed level of search friction $\kappa = \overline{\kappa}$ and varying holding costs $\overline{C}$. For lower values of $\overline{C}$, the model generates multiple equilibria, i.e. one equilibrium with $\beta_{SysSearch} < 0$ and another one with $\beta_{SysSearch} > 0$. Related parameters are: $d = 0.6$, $\overline{\kappa} = 0.05$, $s = 0.625$.

which is equivalent to:

$$\frac{d|\beta_{SysSearch}|}{dCov^o(a, \delta)} > 0, \text{ for } \forall \beta_{SysSearch}. \tag{9}$$

Proof is in Appendix C.1.3. A numerical example is shown in Figure 3.

In Figure 3, the left graph shows that conditional on $\beta_{SysSearch} < 0$, at each level of search friction, the higher the level of initial misallocation is (i.e., the lower $Cov^o(a, \delta)$ is), the lower *absolute* value $\beta_{SysSearch}$ has (i.e., bond yield is less sensitive to change in search friction at each *level* of search friction); the right graph shows that when low-type bond holder's holding cost is low enough, there exist two equilibria, one has $\beta_{SysSearch} < 0$ and the other has $\beta_{SysSearch} > 0$. Conditional on the sign of $\beta_{SysSearch}$, as the level of misallocation increases, the *absolute* derivative of bond yield with respect to search friction always decreases. When the holding cost is high enough, there exists a unique equilibrium with $\beta_{SysSearch} < 0$.

To test the key result $\frac{d}{dCov^o(a,\delta)}|\frac{dP}{d\kappa}| > 0$ in Proposition 3.1, we need to estimate the measure of bond misallocation $Cov^o(a, \delta)$ with expression in (6). Since within the short period, the probability that a δ-type investor holds the bond is $\frac{\phi_1^o(\delta)}{1/2}$ and the probability of not holding is $1 - \frac{\phi_1^o(\delta)}{1/2}$, then $\phi_1^o(\delta)$ is proportional to the δ-type investor's average inventory within the period. Therefore in the data, for any bond-and-period, as long as we are able to compute the estimates of each trader's average inventory position $\widehat{\phi_1}^o(\delta)$ and private valuation $\widehat{\delta}$ for that bond-and-period, we can calculate the sample covariance $\widehat{Cov}^o(\phi_1^o(\delta), \delta)$ and use it as an estimate of the bond's misallocation among traders within that period.

3.2 Estimate of bond misallocation among traders

To estimate each bond's misallocation among traders, we focus on each bond-and-month and define each bond-and-month as a market. Within each market, we need to separately estimate each trader's private valuation for each bond and also her inventory position on the bond. It is important to note that there exist non-trivial gaps between theoretical and empirical moments: (i) private valuation is an assumed measure of traders' *relative preference* for a bond, which is used to rank the bond's all traders by their *relative preference* from low to high. It does not have a directly mapped moment in the data. (ii) it is also difficult to directly estimate the *absolute amount* of each trader's inventory position on each bond, because in data we cannot observe traders' initial bond position. In this section, we propose an estimate of traders' private valuation and we borrow an existing estimate of traders' holding position. Both estimates are not exactly equal to but *monotonically increasing* with their theoretical moments. Then we use these two estimates to estimate the covariance between private valuation and inventory position.

In the data, within each bond-and-month (i.e., market), there are multiple traders trading the bond. For example, there are on average 15 traders buying and/or selling a bond across all bond-and-month, and the maximum value of this number is 256. We also calculate the gap between maximum and minimum transaction prices within each bond-and-month, and the mean and median of this gap are separately 487 bps and 422 bps across all bond-and-month.[44] All above imply that within each market defined by a bond-and-month, traders may have more than two types of private valuation. Therefore, our empirical estimates rely on a setting with multiple types of traders.

[44]In the data, transaction price is reported as a percentage of bond face value.

In the extended setting, traders' private valuation belongs to a continous interval, $\delta \in [\delta_\ell, \delta_h]$, and follows a uniform distribution. At $t = \triangle$, only the bond holder of the highest type δ_h does not pay holding cost, and the amount of holding cost paid by other-type bond holders is monotonically decreasing with private valuation δ. For simplicity, we assume the holding cost of a δ-type trader who holds the bond at maturity equals $c \times (\delta_h - \delta)$, $c > 0$, $\delta \in [\delta_\ell, \delta_h]$, and the highest holding cost satisfies $d - c \times (\delta_h - \delta_\ell) = -d < 0$. At $t = 0$, bond positions are distributed among the traders, following a continuous monotonic (increasing or decreasing) function $\phi_1^o(\delta)$, $\delta \in [\delta_\ell, \delta_h]$. For a specific $\hat{\delta}$, $\phi_1^o(\hat{\delta})$ can be interpreted either as the fraction of $\hat{\delta}$-type traders who hold the bond, or as the probability at which a $\hat{\delta}$-type investor hold the bond,[45] i.e., the $\hat{\delta}$-type investor's average inventory position. Correspondingly, the fraction of $\hat{\delta}$-type traders who do not hold the bond can be denoted as: $\phi_0^o(\delta) = \frac{1}{\delta_h - \delta_\ell} - \phi_1^o(\delta)$. Similar to the simplified setting, we use $Cov(\phi_1^o(\delta), d - c \times (\delta_h - \delta)) \propto Cov(\phi_1^o(\delta), \delta)$, $\delta \in [\delta_\ell, \delta_h]$, as a measure of bond misallocation.

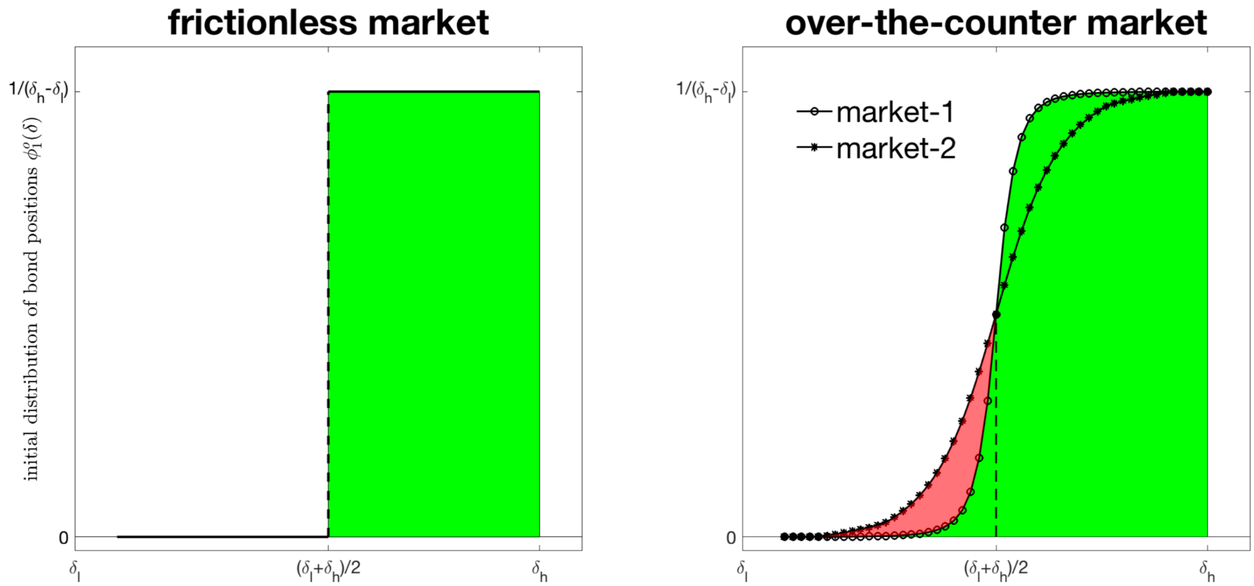
In Figure 4, we give a numerical example which shows different levels of bond misallocation in two over-the-counter (OTC) markets,[46] relative to a counterfactual frictionless market. The area below the function $\phi_1^o(\delta)$, which is equal to $\int_{\delta_\ell}^{\delta_h} \phi_1^o(\delta) d\delta$, represents the total amount of bond positions being held by traders. Suppose in frictionless market, the minimum net payment flow among all bond holders is the middle level $\frac{d - c \times (\delta_h - \delta_\ell) + d - c \times (\delta_h - \delta_h)}{2} = \frac{2d - c \times (\delta_h - \delta_\ell)}{2}$, then in any OTC market, we regard any bond positions, which are held by traders with net payment flows lower than this middle level, as being misallocated. In the right graph of Figure 4, market-1 has relatively fewer positions being misallocated than market-2. The difference in misallocated positions is the red area. Correspondingly, the covariance $Cov(\phi_1^o(\delta), d - c \times (\delta_h - \delta))$ is higher in market-1.

Estimate of private valuation We estimate monthly series of traders' idiosyncratic private valuations using realized transaction prices. The estimator we propose below is monotonically increasing with private valuation across traders, within each bond in each month. By search-and-match setting, the positive gains from searching requires that, for each trader with private valuation δ (i.e. with net payment flow $d - c \times (\delta_h - \delta)$), all her selling (buying) prices are higher (lower) than $d - c \times (\delta_h - \delta)$, as long as her private valuation δ and

[45] Because we normalize the total population measure of all traders to be equal to one, the population measure coincides with the probability measure.

[46] Mapped to data, each market can be regarded as a combination of bond-and-month.

Figure 4. Example of bond misallocation relative to frictionless market (traders with multiple private valuations)



The distributions of bond positions are set such that the measure of bond misallocation $Cov(\phi_1^o(\delta), d - c \times (\delta_h - \delta))$ takes values as follows: in frictionless market, it equals 0.130, in market-1, it equals 0.13, and in market-2, it equals 0.09. The pdf of private valuation is $f(\delta) \equiv \frac{1}{\delta_h - \delta_\ell}$, $\forall \delta \in [\delta_\ell, \delta_h]$. Related parameters are $\delta_\ell = 0$, $\delta_h = 1$, $d = 1.6$, $c = 1$.

bond's fundamentals remain fixed in the period. Then the trader's maximum buying price (minimum selling price) is closest to her net payment flow[47] in her buying (selling) direction. Therefore, a simple average of the two can be used to consistently estimate the trader's net payment flow. Within each bond, across all traders, their net payment flow is monotonically increasing with their idiosyncratic private valuation.

For each pair of bond-and-month, the estimate of traders' private valuation is computed as follows:

$$\hat{\delta}_{i,t}^j = \frac{max\{\widehat{Buy}_{i,n_{i,t}^{j,B}}^j\} + min\{\widehat{Sell}_{i,n_{i,t}^{j,S}}^j\}}{2} \tag{10}$$

where $\{\widehat{Buy}_{i,n_{i,t}^{j,B}}^j\}$ ($\{\widehat{Sell}_{i,n_{i,t}^{j,S}}^j\}$) is the collection of all orthogonalized buying (selling) prices by trader i for bond j in month t, and $n_{i,t}^{j,B}$ ($n_{i,t}^{j,S}$) is the correspondinng number of total

[47]For simplicity, we ignore the discounting process of future net payment flow(s). The "net payment flow" here refers to the summation of a trader's all discounted future net payment flows by holding a bond.

buying (selling) transactions, which include both transactions between two FINRA' member firms and transactions between one FINRA's member firm and one outside non-member firm, in month t. For this estimator, we assume traders' idiosyncratic private valuations remain unchanged within each month.[48] And to control the change in bond's fundamentals, for each buy/sell transaction, we follow Choi and Huh (2019) to use a volume-weighted average transaction price between FINRA's member firms under some restrictions[49] as the current transaction's reference price, and then subtract the reference price from the raw price to obtain an orthogonalized price.

In finite samples, for each trader, the maximum orthogonalized buying price is a downward biased estimate of the trader's net payment flow, while the minimum orthogonalized selling price is an upward biased estimate. Taking the average of two will help cancel out the biases. In small samples with traders' unbalanced buy and sell trades, the levels of upward and downward biases may not be equal. Then to cancel out the biases as much as possible, the weights assigned on the two extreme prices can be adjusted according to the realized number of buy and sell trades. Detailed explanations on the estimator is in Appendix C.2.

Estimate of inventory position We follow the approach in Hansch, Naik, and Viswanathan (1998) to estimate the monthly series of traders' "standardized" inventory position, which is monotonically increasing with real inventory position across traders. We use $Q_{i,t}^j$ to denote the (unobservable) trader i's inventory position in bond j and month t, s.t. $0 \leq t \leq T$, where T is the last month of our sample. We use $q_{i,t}^j$ to denote the corresponding observable signed net trading volume, which is positive (negative) when the trader i increases (shrinks) her inventory position of bond j in month t. With unobservable initial inventory $Q_{i,0}^j$, $Q_{i,t}^j$ satisfies:

$$Q_{i,t}^j = Q_{i,0}^j + \sum_{s=1}^{t} q_{i,s}^j \tag{11}$$

[48] Essentially it is important that within each bond and month, traders' relative preferences for the bond remain unchanged. In other words, if we rank all traders by their private valuation for the bond, from low to high, at the start of the month, the order of traders will remain fixed throughout the whole month.

[49] The restrictions include: the trades used to calculate the reference price are required to be larger than $100,000 in the same bond-day with the current buy/sell trade, and more importantly, these trades can *not* happen within 15 minutes surrounding the current trade.

Then we construct the standardized inventory for each trader i, bond j and month t:

$$I_{i,t}^{j} = \frac{Q_{i,t}^{j} - \bar{Q}_{i}^{j}}{\sigma_{i}^{j}} \tag{12}$$

where $\bar{Q}_{i,t}^{j} = \frac{\sum_{s=0}^{T} Q_{i,s}^{j}}{T+1}$ and $\sigma_{i}^{j} = \sqrt{\frac{\sum_{s=0}^{T}(Q_{i,s}^{j} - \bar{Q}_{i,t}^{j})^2}{T}}$ are the sample mean and standard deviation of the monthly series.[50]

The standardized inventory $I_{i,t}^{j}$ essentially measures by how much the current inventory $Q_{i,t}^{j}$ deviates from the unobserved target level $\bar{Q}_{i,t}^{j}$, and the deviation is scaled by the volatility of the series within each pair of trader i and bond j. By similar derivation in Hansch, Naik, and Viswanathan (1998), this standardization (i) excludes the effect of unobserved initial inventory position $Q_{i,0}^{j}$ after issuance[51], and writes standardized inventory as a linear combination of a series of signed net trading volumes $\{q_{i,s}^{j}\}$; and (ii) controls for differences in risk aversion to guarantee the comparability of inventories across traders (see Friewald and Nagler (2016)).

Estimate of bond misallocation With the estimated monthly series $\{\hat{\delta}_{i,t}^{j}\}_{i,j,t}$ and $\{I_{i,t}^{j}\}_{i,j,t}$, we calculate the series of the covariance between traders' private valuation and inventory position, within each pair of bond j and month t as follows:

$$\widehat{Cov}(I_{i,t}^{j}, \hat{\delta}_{i,t}^{j}) = \frac{1}{N_{trader}^{j,t}} \sum_{i \in Trader_{j,t}} \left(I_{i,t}^{j} - \overline{I}_{t}^{j}\right) * \left(\hat{\delta}_{i,t}^{j} - \overline{\hat{\delta}}_{t}^{j}\right) \tag{13}$$

where $Trader_{j,t}$ is the collection of all traders who completed at least one transaction in bond j on both the buy and sell sides of the market in month t, and $N_{trader}^{j,t}$ is the number of traders within the collection $Trader_{j,t}$; $\overline{\hat{\delta}}_{t}^{j}$ and $\overline{I}_{t}^{j}$ are the simple cross-trader averages of private valuation and inventory position within the collection $Trader_{j,t}$.

[50]For a robustness check, we also follow Friewald and Nagler (2016) to calculate $\bar{Q}_{i,t}^{j}$ and $\sigma_{i,t}^{j}$ only using series of signed trading volumes within the fixed rolling time window $[t, t-R]$. We obtain similar results for our quantitative analysis.

[51]We calculate the series of standardized inventory $\{I_{i,t}^{j}\}$ before dropping bond transactions during a 3-month on-the-run period following issuance.

3.3 Relationship between bond misallocation and liquidity risk

In this section, we show that bonds with a higher level of misallocation are less sensitive to change in search friction, regardless of the direction in which price moves with search friction. This finding supports that: the distribution of traders' idiosyncratic states are related to corporate bond's liquidity risk from search friction.

To verify this relationship, we construct a yearly panel data on bonds' yield-spread loadings on search friction $\beta^j_{SysSearch,y}$ and within-bond-year average monthly covariance between private valuation and inventory position, $\widehat{\overline{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$. In particular, $\beta^j_{SysSearch,y}$ is estimated for bond j which has transactions completed in year y, using bond j's all monthly observations within the time window $[y-2, y]$, which has fixed length of three years. Correspondingly, $\widehat{\overline{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ is constructed as a weighted average of bond j's monthly covariances across all months m_y within the time window $[y-2, y]$. In summary, to construct each point $(\beta^j_{SysSearch,y}, \widehat{\overline{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y}))$ in the yearly panel data, we make use of all the information on realized transactions, market structure, bond fundamentals, and market aggregates, etc, within the most recent three years, for each year y.[52]

We estimate the following regression model, *separately for negative and positive* $\beta^j_{SysSearch,y}$, to verify the relationship between bond's misallocation, $\widehat{\overline{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ and liquidity risk from search friction, $\beta^j_{SysSearch,y}$:

$$\beta^j_{SysSearch,y} = \alpha_0 + \alpha_1 * \widehat{\overline{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y}) + \boldsymbol{\alpha}_2 F^j_y + \phi_J + \eta_y + \epsilon^j_y \tag{14}$$

where the vector F^j_y includes independent variables which potentially cross-sectionally determine bonds' liquidity risk from search friction, such as the weighted averages of bond fundamentals (outstanding amounts, time-to-maturity, credit rating), proportions of transactions only between FINRA's member firms relative to those between one member and one non-member firm, bond liquidity level measured by turnover rate, issuers' financial condition (ROA, leverage ratio, B/M ratio), and level of bond-specific search friction within each time window. The year fixed effect η_y controls all the other unobserved variables which remain cross-sectionally fixed within each time window ending at year y. We also include industry

[52] For summary statistics of constructed monthly series of misallocation and yearly panel data of $\{(\beta^j_{SysSearch,y}, \widehat{\overline{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y}))\}_{j,y}$, see Table 10.

fixed effect ϕ_J to control unobserved variables which are related to bond issuers' industry and remain fixed over time.

The results in Table 3 are consistent with the model prediction that, in cross section, a higher level of misallocation among traders, i.e. a lower value of $\overline{\widehat{Cov}_y}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$, is associated with a lower *absolute value* of liquidity risk from search friction. In other words, a higher aggregate trading need to reallocate a bond's positions among traders makes the bond's price more rigid to change in search friction. For full regression results, see Table 11 and Table 12.

For robustness check, (i) we replace the average bond misallocation, $\overline{\widehat{Cov}_y}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$, within each time window $[y-2, y]$, with the median value of the monthly bond misallocations within the time window, which is denoted as $\widehat{Cov}_y^{Median}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$. This controls the situation that monthly bond misallocation varies significantly across the three years of a time window. The results are consistent with when using $\overline{\widehat{Cov}_y}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ as determinant, see in Table 16 and Table 17 in Internet Appendix. (ii) besides showing how the conditional mean of liquidity risk, $\beta^j_{SysSearch,y}$, varies across different levels of bond misallocation, we also look at how its different quantiles are determined by bond misallocation. The results in Table 4 show that, for $\beta^j_{SysSearch,y}$ with values above the median level (more likely to be positive), their values decrease with bond misallocation (increase with $\overline{\widehat{Cov}_y}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$); for $\beta^j_{SysSearch,y}$ with values below the median level (more likely to be negative), their values increase with bond misallocation (decrease with $\overline{\widehat{Cov}_y}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$).

3.4 Testing the other model predictions

In this section, we test the other two model predictions, which helps validate the channel through which bond misallocation determines bond liquidity risk from search friction. The first one is, bonds with a higher level of initial misallocation among traders are associated with a higher volume of transactions. The second one is, bonds with a higher holding cost for low-type traders are more likely to have a negative value of liquidity risk from search friction, i.e. $\beta_{SysSearch} < 0$.

Table 3. Correlation between *conditional mean* of bond misallocation and liquidity risk from search friction

	Dependent variable $\beta^{j}_{SysSearch,y} < 0$				
	(1)	(2)	(3)	(4)	(5)
$\widehat{Cov}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ $(1,000 \times \%)$	-0.93***	-0.90***	-0.61***	-0.53***	-0.65***
	(-6.82)	(-6.81)	(-4.83)	(-3.36)	(-3.35)
Adj R^2	0.10	0.11	0.19	0.19	0.19
# of Bonds	5013	5013	5013	4012	4012
# of Obs	15028	15028	15028	11837	11837
Industry FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES

	Dependent variable $\beta^{j}_{SysSearch,y} > 0$				
	(1)	(2)	(3)	(4)	(5)
$\widehat{Cov}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ $(1,000 \times \%)$	0.78***	0.74***	0.54***	0.49***	0.48***
	(6.65)	(6.38)	(4.89)	(3.85)	(3.76)
Adj R^2	0.07	0.08	0.16	0.17	0.17
# of Bonds	4337	4337	4337	3471	3471
# of Obs	9775	9775	9775	7726	7726
Industry FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets. Regression (1) only includes bond misallocation as control; Regression (2) adds bond turnover rate as proxy for level of liquidity and the ratio of the number of transactions between FINRA's member firms to the number of transactions between a member firm and a non-member firm; Regression (3) additionally adds bond fundamentals including outstanding amounts, time to maturity and credit rating; Regression (4) additionally adds issuer's financial ratios including B/M ratio, ROA and leverage; Regression (5) additionally adds bond-specific average chain length as proxy for bond-specific *level* of search friction.

3.4.1 Bond misallocation and aggregate trading volume

In the model, we obtain the following relationship between total volume of realized transactions and the initial distribution of bond positions among traders.

$$TotalTrade = 2\lambda^*_1(\ell)\lambda^*_0(h)\phi^o_1(\ell)\phi^o_0(h) \tag{15}$$

Table 4. Correlation between *quantiles* of bond misallocation and liquidity risk from search friction

	Dependent variable $\beta^j_{SysSearch,y}$				
	$\tau = 0.1$	$\tau = 0.2$	$\tau = 0.3$	$\tau = 0.4$	$\tau = 0.5$
$\widehat{\widehat{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ $(1,000 \times \%)$	-1.43***	-0.97***	-0.61***	-0.15*	-0.01
	(-5.06)	(-10.07)	(-8.88)	(-1.71)	(-0.19)
Psuedo R^2	0.34	0.32	0.31	0.30	0.29
	$\tau = 0.6$	$\tau = 0.7$	$\tau = 0.8$	$\tau = 0.9$	$\tau = 0.95$
$\widehat{\widehat{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ $(1,000 \times \%)$	0.08***	0.27***	0.54***	0.88***	1.66***
	(1.55)	(4.45)	(5.50)	(5.75)	(4.47)
Psuedo R^2	0.28	0.28	0.28	0.30	0.31
# of Bonds	4334	4334	4334	4334	4334
# of Obs	19563	19563	19563	19563	19563

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets. All regressions include the following independent variables: bond misallocation, turnover rate, the ratio of the number of transactions between FINRA's member firms to the number of transactions between a member firm and a non-member firm, bond fundamentals (outstanding amounts, time to maturity and credit rating), issuer's financial ratios (B/M ratio, ROA and leverage), and bond-specific average chain length.

where $TotalTrade$ increases with $\phi^o_1(\ell)$, or in other words, increases with the level of bond misallocation. $TotalTrade$ can also be regarded as the total amount of bond positions being reallocated among traders.

We estimate the regression model (16) using yearly panel data, to test the equilibrium condition in (15).

$$Vol^{MM+Non}_{j,y} = \beta_0 + \beta_1 \times \widehat{Cov}^o_y(I^j_{i,y}, \hat{\delta}^j_{i,y}) + \Gamma_1 X^j_y + \phi_J + \eta_y + \epsilon^j_y \tag{16}$$

where $Vol^{MM+Non}_{j,y}$ is the total trading volume including both transactions between two FINRA's member firms (M-to-M) and transactions between one member firm and one non-member firm (M-to-NonM or NonM-to-M) for bond j and year y, $\widehat{Cov}^o_y(I^j_{i,y}, \hat{\delta}^j_{i,y})$ is the within-bond-j covariance between its traders' private valuation and inventory position in the *first* month of year y. $\widehat{Cov}^o_y(I^j_{i,y}, \hat{\delta}^j_{i,y})$ is our most interested independent variable because it measures the *initial* level of bond misallocation in each year. We also incorporate other independent variables, denoted as X^j_y, including the averages of bond's outstanding amounts, time-to-maturity, credit rating, and level of search friction, etc, within each year y. We control industry fixed effect ϕ_J and year fixed effect η_y.

The results in Table 5 shows that, the sign of β_1 is negative, i.e. a lower value of $\widehat{Cov}^o_y(I^j_{i,y}, \hat{\delta}^j_{i,y})$ (a higher initial level of bond misallocation) is associated with a higher value of total trading volume. This is consistent with the model prediction that an initial distribution of bond positions with more positions allocated to lower-type traders motivate all traders to reallocate the positions among themselves. For robustness check, we replace the dependent variable $Vol^{MM+Non}_{j,y}$ with only M-to-M trading volume $Vol^{MM}_{j,y}$ and turnover rate $Turnover_{j,y}$ which is total trading volume divided by bond outstanding amount. The results are still consistent.

3.4.2 Determinants of the sign of $\beta_{SysSearch}$

The low-type traders' holding cost determines whether it is urgent for low-type bond holders to offload their inventory positions to others as quickly as possible. In other words, low-type bond holders' holding cost is closely related to the selling pressure in a bond market. We follow Feldhütter (2012) to use differences in prices for small and large trades as proxy for bond selling pressure, and test its relationship with the sign of $\beta_{SysSearch}$. We consider a regression with the following specification:

$$\mathbb{1}(\beta^j_{SysSearch,y} < 0) = \gamma_0 + \gamma_1 \overline{Dif}^j_y + \boldsymbol{\gamma_2} F^j_y + \phi_J + \eta_y + \epsilon^j_y \tag{17}$$

where $\overline{Dif}^j_y$ is the volume-weighted average price difference between investor sells with a volume of more than $\$1,000,000$ and investor sells with a volume less than $\$100,000$, within the fixed-length time window ending at year y.[53] We also incorporate independent variables

[53] Specifically, we calculate the price difference only for transactions of "non-member firm sold to member firm" and transactions between two member firms. The reason we exclude the transactions where FINRA's member firms sold to putside non-member firms is, we focus on the selling pressure which is sourced from *lower-type* traders. Therefore, by assuming non-member-firm-sellers and member-firm-sellers have relatively lower private valuations, the price difference between large and small trades of those traders is a better proxy for the selling pressure in the market.

Table 5. Correlation between initial level of bond misallocation and total trading volume

	Dependent variable		
	$Vol_{j,y}^{MM+Non}$(\$)	$Vol_{j,y}^{MM}$(\$)	$Turnover_{j,y}$(%)
$\widehat{Cov}_y^o(I_{i,y}^j, \hat{\delta}_{i,y}^j)(1,000 \times \%)$	-2.55**	-0.97***	-0.05***
	(-2.23)	(-2.93)	(-3.05)
$Amtout_y^j$ (\$billion)	1.04e+09***	2.82e+08***	
	(15.57)	(11.12)	
TTM_y^j (thousand days)	2.14	0.50	-0.31***
	(0.53)	(0.43)	(-6.43)
$Rating_y^j$	3.35e+07***	5.61e+06***	0.93***
	(10.53)	(9.32)	(9.43)
$AveChainLength_y^j$	1.34e+08***	5.41e+07***	-3.69***
	(5.95)	(6.44)	(-5.96)
Adj R^2	0.61	0.50	0.05
# of Bonds	5336	5336	5336
# of Obs	24074	24074	24074
Industry FE	YES	YES	YES
Year FE	YES	YES	YES
Bond fundamentals	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets. Standard errors are clustered at the bond level. $Vol_{j,y}^{MM}$ is the volume only including transactions happened between FINRA's member firms. $Turnover_{j,y}$ is the turnover rate (total trading volume divided by outstanding amounts). With $Turnover_{j,y}$ as dependent variable, we exclude bond's outstanding amount from independent variables.

F_y^j same as (14). Finally, we control industry fixed effect ϕ_J and year fixed effect η_y.

Table 6 presents the regression result under a linear probability model (OLS) assumption, together with those under logit and probit model assumptions. The results all indicate a positive value of γ_1, i.e. a higher price difference between large and small trades (a higher selling pressure) is associated with a higher probability of negative value of $\beta_{SysSearch,y}^j$. This is consistent with Proposition 1 in that under a higher value of holding cost $\overline{C}$, there exists a unique equilibrium with negative value of liquidity risk $\beta_{SysSearch,y}^j$.

Table 6. Correlation between selling pressure and the sign of liquidity risk from search friction

	Dependent variable $\mathbb{1}(\beta^j_{SysSearch,y} < 0)$		
	Linear probability	Logit	Probit
$\overline{Dif}^j_y$	0.03**	0.11***	0.07***
	(2.36)	(2.75)	(2.73)
$Amtout^j_y$ (\$trillion)	7.31	31.1	19.8
	(1.08)	(1.21)	(1.25)
TTM^j_y (thousand days)	-0.01***	-0.02***	-0.02***
	(-3.22)	(-3.74)	(-3.74)
$Rating^j_y$	3.07e-03*	0.01***	0.01***
	(1.86)	(2.09)	(2.03)
B/M^j_y	0.07***	0.31***	0.19***
	(6.90)	(7.85)	(7.83)
ROA^j_y	0.02	0.16	0.08
	(0.35)	(0.60)	(0.50)
$Leverage^j_y$	0.19***	0.79***	0.49***
	(6.05)	(6.82)	(6.81)
Adj/Pseudo R^2	0.01	0.01	0.01
# of Bonds	3395	3395	3395
# of Obs	14971	14971	14971

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets. Standard errors are clustered at the bond level.

4 Conclusion

In this paper, we document the cross-sectional heterogeneity in corporate bond's liquidity risk from search friction, which helps explain the cross-section of bond yield spread levels. We propose a measure of bond misallocation among traders, and empirically show that this measure is cross-sectionally correlated with bond's liquidity risk from search friction. In particular, a high (low) level of bond misallocation is associated with a low (high) *absolute* value of liquidity risk from search friction, regardless of the sign of liquidity risk.

Based on a simple search-and-match model, we clarify that bond misallocation determines the absolute value of liquidity risk from search friction, through driving traders' search activity in the decentralized market. In particular, a higher level of misallocation of positions among traders motivate all traders to search at a higher intensity to reallocate the positions among themselves. When search friction increases, although it is more costly to search, the higher level of bond misallocation still motivate traders to maintain a higher search intensity. As a result, for the marginal traders who have strong incentive to offload their holding position (buy bond from others), they do not need to accept a much lower (higher) price than before, in order to maintain the easiness of trade. Therefore, with a higher level of bond misallocation, bond price changes by a lower amount for per unit change in search friction. Similar analysis works for the case of low bond misallocation. The model predictions can be verified by the data.

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Table 7. Summary statistics. Table-A provides mean, standard deviation, 5th, 50th (median) and 95th percentiles of bond-level characteristics. For variables “Offering amount ($million),” “Coupon(%),” and “Maturity (years),” we calculate summary statistics based on bond-level observations; for variables “Amount outstanding($million),” “Credit rating,” “Age (years),” and “Month turnover (%),” we calculate summary statistics based on bond-month observations. “Month turnover” is calculated by dividing bonds’ monthly total trading volume (in units) by bonds’ average amount outstanding for each month. Table-B provides statistics on transaction activity of different sub-markets. “M-to-NonM” indicates transactions in which FINRA’s member firms sell to outside non-member firms; “NonM-to-M” indicates transactions in which FINRA’s member firms buy from outside non-member firms; “M-to-M” indicates transactions between FINRA’s member firms. The sample starts on January 2, 2005 and ends on September 30, 2015.

Table A: bond fundamental characteristics (10760 bonds)

	Mean	Std. dev.	Q5	Q50	Q95
Offering amount ($million)	458.97	577.99	5.74	300.00	1500.00
Coupon(%)	5.72	1.88	2.50	5.65	9.00
Maturity (years)	11.29	7.61	3.28	9.99	30.03
Amount outstanding($million)	499.35	615.95	6.88	350.00	1750.00
Credit rating	8.53 (BBB)	3.94	3.00 (AA)	8.00 (BBB+)	16.00 (B-)
Age (years)	3.70	2.55	0.48	3.17	8.72
Month turnover (%)	6.92	11.42	0.39	3.57	23.76

Table B: transaction activity

	All	M-to-NonM	NonM-to-M	M-to-M
Num of trades (million)	57.62	20.88	15.43	21.31
Total par value($trillion)	27.80	10.57	10.52	6.70
Average par value ($million)	0.48	0.51	0.68	0.31
Average vol (thousand)	482.41	506.25	681.86	314.59
Std. vol (thousand, all bonds)	4.47	5.47	4.46	3.22
Std. vol (thousand, within bond)	1.58	1.62	1.89	0.87

Table 8. Fama-Macbeth regression of yield spread level on factor loadings

Subperiod	*pre-crisis* ($s=1$)	*crisis* ($s=2$)	*post-crisis* ($s=3$)	*regulation* ($s=4$)	*Volcker* ($s=5$)
$\lambda^s_{SysSearch}$ (bps)	-1.21***	-0.89***	-3.65***	-1.13***	-0.89***
	(-3.43)	(-5.70)	(-10.46)	(-6.00)	(-5.18)
$\lambda^s_{SysNetConcen}$ (bps)	0.28**	-0.18***	-0.21**	-0.95***	1.23***
	(2.45)	(-6.55)	(-2.25)	(-10.06)	(9.33)
$\lambda^s_{prearrange}$ (bps)	0.01	-0.36***	0.96***	1.60***	1.60***
	(0.05)	(-3.12)	(6.03)	(10.73)	(6.61)
λ^s_{inv} (bps)	0.35***	0.06***	-0.03	0.20***	-0.18***
	(8.18)	(4.55)	(-0.618)	(4.33)	(-3.87)
$\lambda^s_{blocktrade}$ (bps)	-0.06***	-0.03***	-0.17***	-0.13***	-0.1***
	(-3.20)	(-3.92)	(-14.60)	(-13.5)	(-6.59)
$\lambda^s_{HHItrader}$ (bps)	3.89***	0.40	-0.45	-0.06***	5.66***
	(3.41)	(1.60)	(-0.482)	(-9.67)	(6.91)
Adjusted R^2	0.48	0.51	0.62	0.45	0.42
# of Observations	2371	2468	2517	7078	2958
Bond liquidity and fundamentals	YES	YES	YES	YES	YES
Trade concentration among traders	YES	YES	YES	YES	YES
Segmented market transactions	YES	YES	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets. We exclude bonds with total number of observations smaller or equal to 19 for column (1)-(2) and smaller or equal to 25 for column (3). The reported estimated coefficients are average values taken across all bonds. The corresponding t-statistics are calculated by dividing each reported (average) coefficient value by the standard deviation of the estimates and scaling by the square root of the number of bonds. Specifically, the value of $\beta^j_{SysNetConcen,s}$ is estimated corresponding to when the unit of trader-network concentration is million; the value of $\gamma^j_{1,inv,s}$ is estimated corresponding to when the unit of aggregate inventory position is \$$trillion$; the value of $\gamma^j_{2,HHItrader,s}$ is estimated corresponding to when the unit of bond-level HHI index is thousand.

Table 9. Chain Length and Trade Information (Jan 2005 - Sep 2015)

	Num (thousands)	Vol($1,000)	Markup(%)	Total time elapsed (mins)
CTC	3982.47	1092.33	0.999	10591.89
C(2)TC	1180.52	181.57	1.317	15192.10
C(3)TC	1028.50	155.09	2.102	16253.38
C(4)TC	351.85	55.57	2.334	19404.53
C(5)TC	104.86	112.42	2.112	25066.72
C(6)TC	32.57	64.68	2.374	34231.25
C(7)TC	12.69	125.46	2.272	40545.61

Note: C(i)TC means there are i traders on the chain; Vol($1,000) is the average trading volume per chain calculated for each length throughout the whole sample period; Markup(%) is the average total markup per chain calculated for each length throughout the whole sample period; Total time(mins) is the average total time gap per chain calculated for each length throughout the whole sample period; We record an intermediation chain as being pre-arranged if its total time is shorter than 1 minute.

Table 10. Summary statistics on monthly and yearly series of bond misallocation. This table provides mean, standard deviation, 5th, 50th (median) and 95th percentiles of monthly series of bond misallocation $\{\widehat{Cov}(I^j_{i,t}, \hat{\delta}^j_{i,t})\}_{j,t}$, yearly series of bond misallocation $\{\overline{\widehat{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})\}_{j,y}$ and $\{\widehat{Cov}_y^{Median}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})\}_{j,y}$, and yearly series of liquidity risk from search friction $\{\beta^j_{SysSearch,y}\}_{j,y}$, among which $\widehat{Cov}_y^{Median}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ denotes the median value of the monthly misallocations of bond j within the time window ending at year y. The unit of bond misallocation is $\$ \times \%$, because in data, transaction volume is reported in par value and transaction price is reported as percentage of face value.

	Mean	Std. dev.	Q5	Q50	Q95
$\widehat{Cov}(I^j_{i,t}, \hat{\delta}^j_{i,t})$ ($\$1 \times \%$)	-4.20	417.18	-359.41	0.00	340.06
$\overline{\widehat{Cov}}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ ($\$1 \times \%$)	29.94	412.20	-448.73	10.77	569.88
$\widehat{Cov}_y^{Median}(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ ($\$1 \times \%$)	13.55	84.59	-94.15	7.026	144.93
$\beta^j_{SysSearch,y}$	-1.91	7.75	-16.03	-0.84	8.54

Table 11. Correlation between bond misallocation and liquidity risk from search friction

	Dependent variable:				
	$\beta^j_{SysSearch,y} < 0$				
	(1)	(2)	(3)	(4)	(5)
$\widehat{Cov}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ $(1,000 \times \%)$	-0.93***	-0.90***	-0.61***	-0.53***	-0.65***
	(-6.82)	(-6.81)	(-4.83)	(-3.36)	(-3.35)
$turnover^j_y$ (%)		-1.87***	-0.80***	-0.50***	-0.50***
		(-7.58)	(-3.39)	(-2.08)	(-2.11)
$Num\_MM^j_y$ (thousand)		-1.67***	-0.68***	-0.10	-0.10
		(-8.68)	(-3.60)	(-0.52)	(-0.54)
$Num\_Non^j_y$ (thousand)		1.49***	0.54***	0.21	0.21
		(11.17)	(3.87)	(1.50)	(1.52)
$Amtout^j_y$ ($billion)			0.74***	0.78***	0.75***
			(7.32)	(7.39)	(6.69)
TTM^j_y (thousand days)			0.11***	0.10***	0.10***
			(5.59)	(4.91)	(4.96)
$Rating^j_y$			-0.66***	-0.58***	-0.58***
			(-36.11)	(-26.59)	(-26.41)
B/M^j_y				-1.96***	-1.94***
				(-15.09)	(-14.75)
ROA^j_y				1.20	1.16
				(1.29)	(1.25)
$Leverage^j_y$				-2.61***	-2.58***
				(-5.84)	(-5.75)
$AveChainLength^j_y$					-0.12
					(-0.77)
Adj R^2	0.10	0.11	0.19	0.19	0.19
# of Bonds	5013	5013	5013	4012	4012
# of Obs	15028	15028	15028	11837	11837
Industry FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets. All the following variables are weighted averages within the time window $[1, y]$: $turnover^j_y$ is turnover rate which is the ratio of total trading volume to total outstanding amount; $Num\_MM^j_y$ is the number of transactions happened between FINRA's member firms, and $Num\_Non^j_y$ is the number of transactions happened between a member and a non-member firm; bond fundamentals include outstanding amount $Amtout^j_y$, time to maturity TTM^j_y, and credit rating $Rating^j_y$.

Table 12. Correlation between bond misallocation and liquidity risk from search friction

	Dependent variable:				
	$\beta^j_{SysSearch,y} > 0$				
	(1)	(2)	(3)	(4)	(5)
$\widehat{Cov}_y(I^j_{i,m_y}, \hat{\delta}^j_{i,m_y})$ $(1,000 \times \%)$	0.78***	0.74***	0.54***	0.49***	0.48***
	(6.65)	(6.38)	(4.89)	(3.85)	(3.76)
$turnover^j_y$ (%)		0.92***	0.34***	1.10***	1.15***
		(5.78)	(2.22)	(4.35)	(4.51)
$Num\_MM^j_y$ (thousand)		0.98***	0.32*	0.09	0.10
		(5.13)	(1.73)	(0.46)	(0.54)
$Num\_Non^j_y$ (thousand)		-1.01***	-0.31**	-0.19	-0.21
		(-8.50)	(-2.49)	(-1.45)	(-1.58)
$Amtout^j_y$ ($billion)			-0.66***	-0.61***	-0.52***
			(-6.91)	(-5.99)	(-4.83)
TTM^j_y (thousand days)			-0.09***	-0.08***	-0.08***
			(-5.25)	(-4.33)	(-4.62)
$Rating^j_y$			0.43***	0.36***	0.37***
			(27.26)	(19.20)	(19.39)
B/M^j_y				1.20***	1.15***
				(9.40)	(8.91)
ROA^j_y				0.20	0.26
				(0.25)	(0.32)
$Leverage^j_y$				2.13***	2.00***
				(5.50)	(5.13)
$AveChainLength^j_y$					0.45***
					(2.77)
Adj R^2	0.07	0.08	0.16	0.17	0.17
# of Bonds	4337	4337	4337	3471	3471
# of Obs	9775	9775	9775	7726	7726
Industry FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are in brackets. All the following variables are weighted averages within the time window $[1, y]$: $turnover^j_y$ is turnover rate which is the ratio of total trading volume to total outstanding amount; $Num\_MM^j_y$ is the number of transactions happened between FINRA's member firms, and $Num\_Non^j_y$ is the number of transactions happened between a member and a non-member firm; bond fundamentals include outstanding amount $Amtout^j_y$, time to maturity TTM^j_y, and credit rating $Rating^j_y$.

Figure 5. Liquidity risk of bonds in different industries

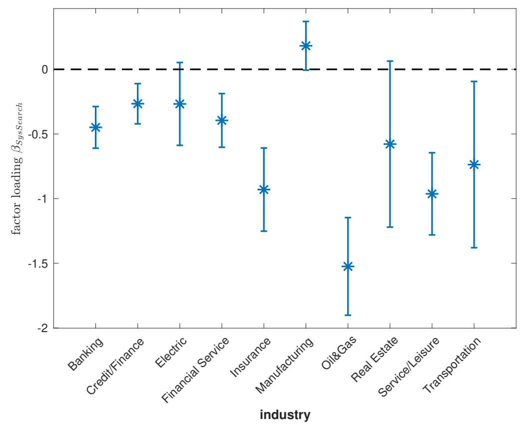
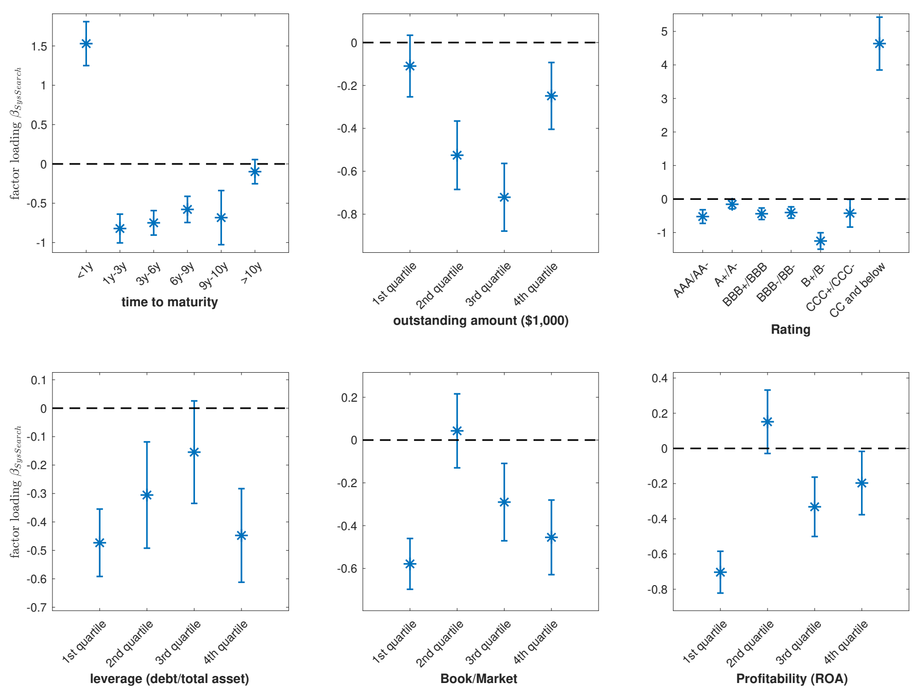


Figure 6. Liquidity risk of bonds classified into different groups



Note: For each group of bonds, we plot the point estimate of factor loading $\beta_{SysSearch}$ together with its 95% confidence interval.

Figure 7. Distribution of bond-specific yield spread loading on systemic search friction (11176 bonds)

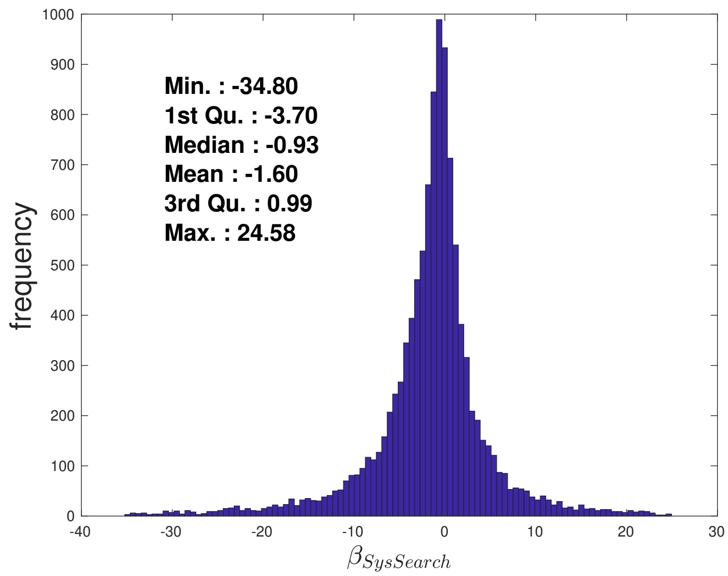
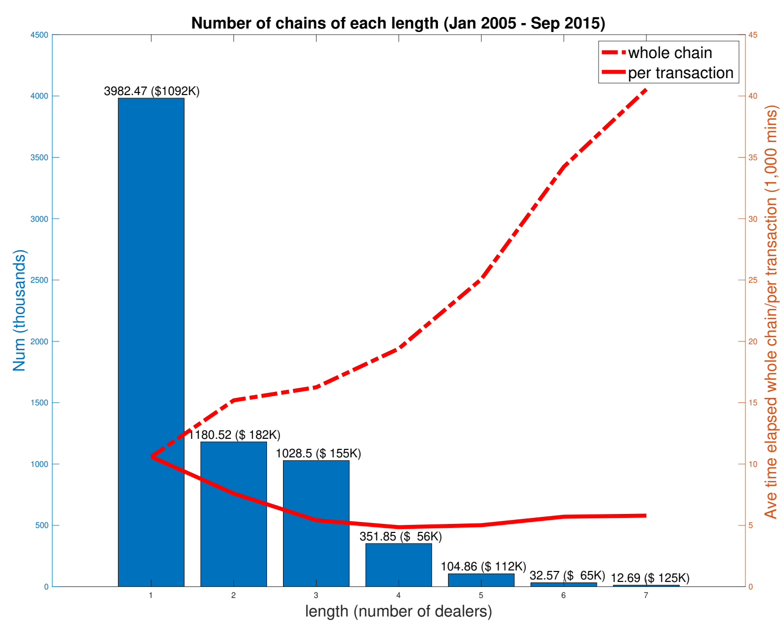


Figure 8. Summary on identified intermediation chains



Note: bar height represents the number of identified intermediation chains for each length. The number in brackets represents the average trading volume across chains for each length.

Appendices

A Identify pre-arranged transactions

In this paper, the "pre-arranged" transactions we consider include agency(or agency-like) transactions and risk-principal transactions. For **agency transactions**: traders behave as "match-makers" to pre-arrange transactions between buyers and sellers, and do not hold bonds in their inventories. Each pair of agency-like transactions have the same price and volume in opposite directions, and happen at very close or exactly the same execution time(s). In the data, we identify agency transactions by two approaches: [1] by the fields "Buyer/Seller Capacity" with value as "Agency"; and [2] we look for pairs of two trades (with the Capacity fields not as "Agency") in a given bond with the same volume by the same trader but in opposite directions, and take place within 15 minutes of each other. As FINRA requires that reports need to be submitted within 15 mins after the transactions happened, this approach will identify those pairs with one buy and one sell transactions which do not have exactly the same execution time. The difference in execution time could be due to reporting errors, so that it is still very likely that the traders conducting the matched buy and sell transactions do not take any inventory risk. For **riskless-principal transactions**: traders temporarily take bond positions in their inventories but without taking any inventory risk. In the data, we identify riskless-principal transactions through matching buy and sell transactions with the fields of "Buyer/Seller Capacity" as "Principal", and conducted by the same trader and with the same volume, price and execution time.

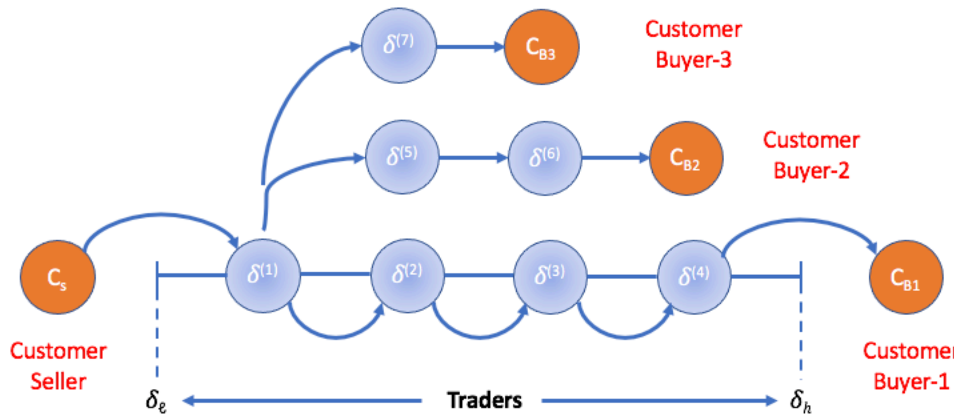
B Identification of intermediation chains

The matching algorithm to construct intermediation chains is an extension of the algorithms in Hollifield, Neklyudov, and Spatt (2017) and Li and Schürhoff (2014). Note that to be consistent with these two papers, *only in this section*, we call FINRA's member firm as "trader" and denote it as "T", and we call outside non-member firm as "customer" and denote it as "C". Similarly, the intermediation chains start from customer-sell-to-trader trades and end at trader-sell-to-customer trades. We also use the first-in-first-out(FIFO) matching algorithm to look for the next trades for each incomplete chain. The main difference is, we only allow the split matching in the first round of the loop. After the first round, we track a fixed par amount of a bond until finding the final customer buyer.

Each intermediation chain starts from a trade that a customer C_s sells some amount of a bond to a trader T_1. We then look for the next trade completed by trader T_1 selling to a customer or another trader within a calendar time window from -1 day to +30 days around the initial C_s-sells-to-T_1 trade. The initial trade is then followed by a trade that the trader T_1 sells the same amount (of the same bond) either to a customer C_e or to another trader T_2. In the first case of selling-to-C_e, the current intermediation chain ends and it is recorded as a CTC chain, that is, there is one trader on the chain; In the second case of selling-to-T_1, the current intermediation chain is not ended and is temporarily recorded as an incomplete chain CTT. We continue looking for trades completed by trader T_2 selling to a customer or another trader within the same calendar time window. This process will continue until finding a trader-sell-to-customer trade of the same bond in same par amount.

We only consider "split matching" in the first round of loop in the sense that, given the initial C_s-sell-to-T_1 trade, we look for a trade with T_1 as the seller of the same bond and with the shortest time gap to the initial trade. Suppose the initial trade has par amount Q_1 and the next closest trade is "trader T_1 sells Q_2 of the same bond to a trader T_2". Then if $Q_1 > Q_2$, that is, the initial trade has larger par amount than the second trade, we split Q_1 into two pieces Q_2 and $Q_1 - Q_2$, and we record a new incomplete chain CTT with par amount Q_2 and put the remaining par amount $Q_1 - Q_2$ (sold by C_s to T_1) back to the pile of initial customer-to-trader trades to be used to initiate new intermediation chains; If $Q_1 < Q_2$, similarly, we split Q_2 into two pieces Q_1 and $Q_2 - Q_1$, and we record a new incomplete chain CTT with par amount Q_1 and put the remaining par amount $Q_2 - Q_1$ (sold by T_1 to T_2) back to the pile of candidate inter-trader trades that will be used to generate more intermediation chains. After the first round of the loop, for all incomplete chains CTT, we restrict that all matched trades on the same intermediation chain after the first round need to have exacty the same par amounts. Same as Li and Schürhoff (2014), we allow for up to 7 traders on an intermediation chain. Figure 9 shows the "split matching" in the first round. Table 9 reports the average trading information of intermediation chains of each length.

Figure 9. Split matching in constructing intermediation chains



Note: $\delta^{(i)}$ denotes the value of the ith trader's private valuation. In this example, the initial customer-sell-to-trader-$\delta^{(1)}$ transaction finally generates three identified intermediation chains through splitting. The intermediation chains ending at Customer-Buyer-1, Customer-Buyer-2 and Customer-Buyer-3 correspondingly have lengths as four traders, two traders and one trader.

C Model

C.1 Proof of Proposition 1

C.1.1 solve equilibrium components under restriction $d - \overline{C} = -d$

Since we already have $\lambda_0^*(\ell) = \lambda_1^*(h) = 0$, the traders' problem (3) will reduce to low-type owner's problem $P_{1,\ell}$ and high-type nonowner's problem $P_{0,h}$ as follows[54]:

$$P_{1,\ell}: \quad U_1(\ell) = \max_{\lambda_1(\ell)\geq 0} \quad \left[1 - e^{-2\lambda_1(\ell)\lambda_0^*(h)\phi_0^o(h)\triangle}\right] \times P + e^{-2\lambda_1(\ell)\lambda_0^*(h)\phi_0^o(h)\triangle} \times \left(d - \overline{C}\right) - \kappa\lambda_1(\ell)^2\triangle \tag{18}$$

$$= \max_{\lambda_1(\ell)\geq 0} \quad 2\lambda_1(\ell)\lambda_0^*(h)\phi_0^o(h)\triangle \times P + (1 - 2\lambda_1(\ell)\lambda_0^*(h)\phi_0^o(h)\triangle) \times \left(d - \overline{C}\right) - \kappa\lambda_1(\ell)^2\triangle$$

[54] We ignore the discount rate r since it does not affect solutions to traders' problem.

$$P_{0,h}: \quad U_0(h) = \max_{\lambda_0(h)\geq 0} \quad \left[1 - e^{-2\lambda_0(h)\lambda_1^*(\ell)\phi_1^o(\ell)\triangle}\right] \times (d-P) - \kappa\lambda_0(h)^2\triangle \tag{19}$$
$$= \max_{\lambda_0(h)\geq 0} \quad 2\lambda_0(h)\lambda_1^*(\ell)\phi_1^o(\ell)\triangle \times (d-P) - \kappa\lambda_0(h)^2\triangle$$

The optimality conditions of problems $P_{1,\ell}$ and $P_{0,h}$ are as follows:

$$\lambda_1^*(\ell) = \frac{\lambda_0^*(h)\phi_0^o(h)(P-d+\overline{C})}{\kappa} \tag{20}$$

$$\lambda_0^*(h) = \frac{\lambda_1^*(\ell)\phi_1^o(\ell)(d-P)}{\kappa} \tag{21}$$

$\Longrightarrow$

$$\frac{\lambda_1^*(\ell)}{\lambda_0^*(h)} = \frac{\phi_0^o(h)(P-d+\overline{C})}{\kappa} = \frac{\kappa}{\phi_1^o(\ell)(d-P)} \tag{22}$$

$\Longrightarrow$

equilibrium price P solves:

$$\kappa^2 = \phi_1^o(\ell)\phi_0^o(h)(P-d+\overline{C})(d-P) \tag{23}$$

$\Longrightarrow$

$$P^2 - (d+d-\overline{C})P + d(d-\overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)} = 0 \tag{24}$$

With restrictions $0 < P < d$, $d - \overline{C} = -d < 0$, we have the unique transaction price

$$P = \frac{2d - \overline{C} + \sqrt{(2d-\overline{C})^2 - 4(d(d-\overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)})}}{2} \tag{25}$$

which requires $(2d-\overline{C})^2 - 4(d(d-\overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)}) = \overline{C}^2 - \frac{4\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)} > 0$.

With normalization $U_1(\ell) = 0$,

$$U_1(\ell) = d - \overline{C} + \kappa\lambda_1^{*2}(\ell)\triangle = 0 \tag{26}$$

$\Longrightarrow$

$$\lambda_1^*(\ell) = \sqrt{\frac{d}{\kappa\triangle}} \tag{27}$$

Then the other equilibrium components are calculated as follows:

$$\lambda_0^*(h) = \frac{\lambda_1^*(\ell)\phi_1^o(\ell)(d-P)}{\kappa} = \frac{\sqrt{\frac{d}{\kappa\triangle}}\phi_1^o(\ell)(d-P)}{\kappa} \tag{28}$$

$$U_0(h) = \kappa\lambda_0^*(h)^2\triangle = \frac{d(\phi_1^o(\ell))^2(d-P)^2}{\kappa^2} \tag{29}$$

(a) $\frac{dP}{d\kappa} < 0$ **and** $\frac{d}{d\phi_1^o(\ell)}|\frac{dP}{d\kappa}| < 0$**:**

By (25),

$$\frac{dP}{d\kappa} = \frac{1}{\sqrt{A}} \times \frac{-8\kappa}{\phi_1^o(\ell)\phi_0^o(h)} < 0 \tag{30}$$

where $A = (2d-\overline{C})^2 - 4(d(d-\overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)})$.

Then by $\phi_0^o(h) = \frac{1}{2} - s + \phi_1^o(\ell)$, $\frac{d\left(\phi_1^o(\ell)\phi_0^o(h)\right)}{d\phi_1^o(\ell)} = 2\phi_1^o(\ell) + \frac{1}{2} - s > 0$, we have,

$$\begin{aligned} \frac{d}{d\phi_1^o(\ell)}|\frac{dP}{d\kappa}| &= 8\kappa \times \frac{d}{d\phi_1^o(\ell)}\left(\frac{1}{\sqrt{A}} \times \frac{1}{\phi_1^o(\ell)\phi_0^o(h)}\right) \\ &= \frac{-8\kappa}{A \times (\phi_1^o(\ell)\phi_0^o(h))^2}\left(\phi_1^o(\ell)\phi_0^o(h)\frac{2\kappa^2}{\sqrt{A}(\phi_1^o(\ell)\phi_0^o(h))^2}\underbrace{\frac{d\left(\phi_1^o(\ell)\phi_0^o(h)\right)}{d\phi_1^o(\ell)}}_{>0} + \sqrt{A}\underbrace{\frac{d\left(\phi_1^o(\ell)\phi_0^o(h)\right)}{d\phi_1^o(\ell)}}_{>0}\right) \\ &< 0 \end{aligned} \tag{31}$$

(b) $\frac{d}{dCov^o(a,\delta)}|\frac{dP}{d\kappa}| > 0$**:**

By (6),

$$\frac{dCov^o(a,\delta)}{d\phi_1^o(\ell)} = -\overline{C} \tag{32}$$

With (31), we have,

$$\frac{d}{dCov^o(a,\delta)}|\frac{dP}{d\kappa}| = \frac{d\phi_1^o(\ell)}{dCov^o(a,\delta)} \times \frac{d}{d\phi_1^o(\ell)}|\frac{dP}{d\kappa}| = \frac{1}{-\overline{C}} \times \frac{d}{d\phi_1^o(\ell)}|\frac{dP}{d\kappa}| > 0 \tag{33}$$

C.1.2 equilibrium under new restrictions $d \geq \frac{2\kappa}{\sqrt{\phi_1^o(\ell)\phi_0^o(h)}}$ and $d < \overline{C} \leq 2d$

Again by equilibrium conditions, price P solves:

$$P^2 - (d + d - \overline{C})P + d(d - \overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)} = 0 \tag{34}$$

Given $\overline{C}^2 > \frac{4\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)}$, without restricting $P > 0$, solutions to (34) are P_1 and P_2:

$$P_1 = \frac{2d - \overline{C} + \sqrt{(2d - \overline{C})^2 - 4(d(d - \overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)})}}{2} \tag{35}$$

$$P_2 = \frac{2d - \overline{C} - \sqrt{(2d - \overline{C})^2 - 4(d(d - \overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)})}}{2} \tag{36}$$

If $d < \overline{C} \leq d + \frac{\kappa^2}{d\phi_1^o(\ell)\phi_0^o(h)}$**:** $0 < d(d - \overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)} < \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)}$, then both P_1 and P_2 are positive. When equilibrium price is P_1, based on same proofs in section C.1.1, we have $\frac{dP_1}{d\kappa} < 0$, $\beta^{P_1}_{SysSearch} = \frac{1}{0.009(1+P)^2} \times \frac{dP_1}{d\kappa} < 0$ and $\frac{d}{dCov^o(a,\delta)}|\frac{dP_1}{d\kappa}| > 0$. When equilibrium price is P_2, we have,

$$\frac{dP_2}{d\kappa} = \frac{1}{\sqrt{A}} \times \frac{8\kappa}{\phi_1^o(\ell)\phi_0^o(h)} > 0 \tag{37}$$

$\Longrightarrow$

$$\begin{aligned} \frac{d}{d\phi_1^o(\ell)}|\frac{dP_2}{d\kappa}| &= \frac{d}{d\phi_1^o(\ell)}\left(\frac{dP_2}{d\kappa}\right) \\ &= 8\kappa \times \frac{d}{d\phi_1^o(\ell)}\left(\frac{1}{\sqrt{A}} \times \frac{1}{\phi_1^o(\ell)\phi_0^o(h)}\right) \\ &< 0 \qquad \text{(same as (31))} \end{aligned} \tag{38}$$

$\Longrightarrow$

$$\frac{d}{dCov^o(a,\delta)}|\frac{dP_2}{d\kappa}| = \frac{d\phi_1^o(\ell)}{dCov^o(a,\delta)} \times \frac{d}{d\phi_1^o(\ell)}|\frac{dP_2}{d\kappa}| > 0, \qquad \text{since } \frac{dCov^o(a,\delta)}{d\phi_1^o(\ell)} = -\overline{C} < 0 \tag{39}$$

If $d + \frac{\kappa^2}{d\phi_1^o(\ell)\phi_0^o(h)} < \overline{C} \leq 2d$**:** $-d^2 + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)} < d(d - \overline{C}) + \frac{\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)} < 0$, then $P_1 > 0$ and $P_2 < 0$, therefore only P_1 can be the equilibrium price.

C.1.3 proof of inequality (8)

By (7),

$$\beta^{P_i}_{SysSearch} = \frac{1}{0.009(1+P_i)^2} \times \frac{dP_i}{d\kappa}, \quad \text{for } i = 1, 2 \tag{40}$$

$\Longrightarrow$

$$\begin{aligned} \frac{d\beta^{P_i}_{SysSearch}}{dCov^o(a,\delta)} &= \frac{\frac{d(\frac{dP_i}{d\kappa})}{dCov^o(a,\delta)}(1+P_i)^2 - 2\frac{dP_i}{d\kappa}(1+P_i)\frac{dP_i}{dCov^o(a,\delta)}}{0.009(1+P_i)^4} \\ &= \frac{\frac{d(\frac{dP_i}{d\kappa})}{d\phi^o_1(\ell)}(1+P_i) - 2\frac{dP_i}{d\kappa}\frac{dP_i}{d\phi^o_1(\ell)}}{0.009(1+P_i)^3} \times \frac{d\phi^o_1(\ell)}{dCov^o(a,\delta)} \end{aligned} \tag{41}$$

(a) For $i = 1$, $\beta^{P_1}_{SysSearch} = \frac{1}{0.009(1+P_1)^2} \times \frac{dP_1}{d\kappa} = \frac{1}{0.009(1+P_1)^2} \times \frac{1}{\sqrt{A}} \times \frac{-8\kappa}{\phi^o_1(\ell)\phi^o_0(h)} < 0$, then

$$\begin{aligned} \frac{d\beta^{P_1}_{SysSearch}}{dCov^o(a,\delta)} &= \frac{\frac{d(\frac{dP_1}{d\kappa})}{d\phi^o_1(\ell)}(1+P_1) - 2\frac{dP_1}{d\kappa}\frac{dP_1}{d\phi^o_1(\ell)}}{0.009(1+P_1)^3} \times \frac{d\phi^o_1(\ell)}{dCov^o(a,\delta)} \\ &= \left\{\left(\frac{2\kappa^2}{\sqrt{A}(\phi^o_1(\ell)\phi^o_0(h))} + \sqrt{A}\right)(1+P_1) + \frac{8\kappa^2}{\phi^o_1(\ell)\phi^o_0(h)}\right\} \times \frac{8\kappa}{A \times (\phi^o_1(\ell)\phi^o_0(h))^2} \\ &\times \frac{d\left(\phi^o_1(\ell)\phi^o_0(h)\right)}{d\phi^o_1(\ell)} \times \frac{\frac{d\phi^o_1(\ell)}{dCov^o(a,\delta)}}{0.009(1+P_1)^3} \\ &< 0 \qquad \left(\text{because } \frac{d\phi^o_1(\ell)}{dCov^o(a,\delta)} < 0 \text{ and } \frac{d\left(\phi^o_1(\ell)\phi^o_0(h)\right)}{d\phi^o_1(\ell)} > 0\right) \end{aligned}$$

(b) For $i = 2$, $\beta^{P_2}_{SysSearch} = \frac{1}{0.009(1+P_2)^2} \times \frac{dP_2}{d\kappa} = \frac{1}{0.009(1+P_2)^2} \times \frac{1}{\sqrt{A}} \times \frac{8\kappa}{\phi^o_1(\ell)\phi^o_0(h)} > 0$. Firstly with the additional restrictions $\kappa^2 \le \frac{1-s}{16}$ and $\frac{s-\frac{1}{2}+\sqrt{(\frac{1}{2}-s)^2+32\kappa^2}}{2} \le \phi^o_1(\ell) \le \frac{1}{2}$, we have,

$$\begin{aligned} (\phi^o_1(\ell))^2 + (\frac{1}{2}-s)\phi^o_1(\ell) &\ge \left(\frac{s-\frac{1}{2}+\sqrt{(\frac{1}{2}-s)^2+32\kappa^2}}{2}\right)^2 + (\frac{1}{2}-s)\frac{s-\frac{1}{2}+\sqrt{(\frac{1}{2}-s)^2+32\kappa^2}}{2} \\ &= 8\kappa^2 + \frac{1}{4}\left(s-\frac{1}{2}+\frac{1}{2}-s\right)^2 \\ &= 8\kappa^2 \end{aligned} \tag{42}$$

where the first inequality is because the function $(\phi_1^o(\ell))^2 + (\frac{1}{2} - s)\phi_1^o(\ell)$ is strictly increasing at the point $\frac{s-\frac{1}{2}+\sqrt{(\frac{1}{2}-s)^2+32\kappa^2}}{2}$, regardless of whether $s < \frac{1}{2}$ or $s > \frac{1}{2}$. then we have,

$$(\phi_1^o(\ell))^2 + (\frac{1}{2} - s)\phi_1^o(\ell) = \phi_1^o(\ell)\left(\frac{1}{2} - s + \phi_1^o(\ell)\right) = \phi_1^o(\ell)\phi_0^o(h) \geq 8\kappa^2 \tag{43}$$

Then as for the derivative of liquidity risk from search friction with respect to bond mis-allocation for $i = 2$, we have,

$$\begin{aligned}
\frac{d\beta_{SysSearch}^{P_2}}{dCov^o(a,\delta)} &= \frac{\frac{d(\frac{dP_2}{d\kappa})}{d\phi_1^o(\ell)}(1+P_2) - 2\frac{dP_2}{d\kappa}\frac{dP_2}{d\phi_1^o(\ell)}}{0.009(1+P_2)^3} \times \frac{d\phi_1^o(\ell)}{dCov^o(a,\delta)} \\
&= \left\{\left(\frac{2\kappa^2}{\sqrt{A}(\phi_1^o(\ell)\phi_0^o(h))} + \sqrt{A}\right)(1+P_2) - \frac{8\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)}\right\} \times \underbrace{\frac{-8\kappa}{A \times (\phi_1^o(\ell)\phi_0^o(h))^2}}_{<0} \\
&\quad \times \frac{d\left(\phi_1^o(\ell)\phi_0^o(h)\right)}{d\phi_1^o(\ell)} \times \frac{\frac{d\phi_1^o(\ell)}{dCov^o(a,\delta)}}{0.009(1+P_2)^3} \\
&\geq \left\{2 \times \left(\sqrt{\frac{2\kappa^2}{\sqrt{A}(\phi_1^o(\ell)\phi_0^o(h))} \times \sqrt{A}}\right) \times (1+P_2) - \frac{8\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)}\right\} \times \underbrace{\frac{-8\kappa}{A \times (\phi_1^o(\ell)\phi_0^o(h))^2}}_{<0} \\
&\quad \times \frac{d\left(\phi_1^o(\ell)\phi_0^o(h)\right)}{d\phi_1^o(\ell)} \times \frac{\frac{d\phi_1^o(\ell)}{dCov^o(a,\delta)}}{0.009(1+P_2)^3} \\
&> \left\{2 \times \left(\sqrt{\frac{2\kappa^2}{\sqrt{A}(\phi_1^o(\ell)\phi_0^o(h))} \times \sqrt{A}}\right) - \frac{8\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)}\right\} \times \underbrace{\frac{-8\kappa}{A \times (\phi_1^o(\ell)\phi_0^o(h))^2}}_{<0} \\
&\quad \times \frac{d\left(\phi_1^o(\ell)\phi_0^o(h)\right)}{d\phi_1^o(\ell)} \times \frac{\frac{d\phi_1^o(\ell)}{dCov^o(a,\delta)}}{0.009(1+P_2)^3} \\
&= \left\{\sqrt{\frac{8\kappa^2}{(\phi_1^o(\ell)\phi_0^o(h))}} - \frac{8\kappa^2}{\phi_1^o(\ell)\phi_0^o(h)}\right\} \times \underbrace{\frac{-8\kappa}{A \times (\phi_1^o(\ell)\phi_0^o(h))^2}}_{<0} \\
&\quad \times \frac{d\left(\phi_1^o(\ell)\phi_0^o(h)\right)}{d\phi_1^o(\ell)} \times \frac{\frac{d\phi_1^o(\ell)}{dCov^o(a,\delta)}}{0.009(1+P_2)^3} \\
&> 0 \qquad \text{(because of (43), } \frac{d\phi_1^o(\ell)}{dCov^o(a,\delta)} < 0 \text{ and } \frac{d\left(\phi_1^o(\ell)\phi_0^o(h)\right)}{d\phi_1^o(\ell)} > 0\text{)}
\end{aligned}$$

C.2 Estimate of traders' private valuations

In the one-period search-and-match model, we denote bond holder' value function as $V_1(\delta) = d - c \times (\delta_h - \delta)$ and non-holder' value function as $V_0(\delta) = 0$, for $\delta \in [\delta_\ell, \delta_h]$. Then we define traders' marginal valuation of the bond is $\triangle V(\delta) = V_1(\delta) - V_0(\delta) = d - c \times (\delta_h - \delta)$, for $\delta \in [\delta_\ell, \delta_h]$, which measures how much compensation each trader requires for giving up holding one unit of the bond. In the bilateral search environment, when two traders (suppose one holds one unit of the bond and the other does not hold any position) with different private valuations meet, transaction only happens when the bond holder's private valuation is lower than that of the non-holder. Since net payment flow conditional on holding the bond is monotonic with private valuation, the realized transaction price will lie in between the net payment flows of the two traders.[55] For a trader with a type $\delta \in [\delta_\ell, \delta_h]$, her transaction price with another trader with a type $\delta' \in [\delta_\ell, \delta_h]$ is:

$$P(\delta, \delta') = \omega_1 \triangle V(\delta) + \omega_2 \triangle V(\delta') \tag{44}$$

where the unknown weights $0 < \omega_1, \omega_2 < 1$ satisfies $\omega_1 + \omega_2 = 1$. Whether $P(\delta, \delta')$ is a selling or buying price depends on whether the trader δ "holds the bond and search on her sell side" or "does not hold the bond and search on her buy side".

For transactions happening on the sell side of the trader δ, since $\triangle V(\delta') > \triangle V(\delta)$(otherwise the transaction would not happen), if it is possible for trader δ to meet a continuum of other traders, the lowest selling price is exactly equal to $\triangle V(\delta) = d - c \times (\delta_h - \delta)$. Vice versa, on the buy side of the trader δ, since $\triangle V(\delta') < \triangle V(\delta)$, the highest buying price is exactly equal to $\triangle V(\delta) = d - c \times (\delta_h - \delta)$. Again since $\triangle V(\delta) = d - c \times (\delta_h - \delta)$ is monotonically increasing with private valuation δ, we construct the following consistent estimator as a proxy for traders' private valuation δ:

$$\hat{\delta}^j_{i,t} = \frac{max\{\widehat{Buy}^j_{i,n^{j,B}_{i,t}}\} + min\{\widehat{Sell}^j_{i,n^{j,S}_{i,t}}\}}{2} \tag{45}$$

where $\{\widehat{Buy}^j_{i,n^{j,B}_{i,t}}\}$ ($\{\widehat{Sell}^j_{i,n^{j,S}_{i,t}}\}$) is the collection of all orthogonalized buying (selling) prices by trader i for bond j in month t, and $n^{j,B}_{i,t}$ ($n^{j,S}_{i,t}$) is the correspondinng number of total buying (selling) transactions in month t.

[55] Here for simplicity, we ignore the process of formation of the transaction price. The price could be generated by a bargaining process or a trader makes a take-it-or-leave-it offer to the other. The reason we pay attention to each trader's series of realized transaction prices is that, the maximum and minimum of a trader's buying and selling prices can separately provide a lower and an upper bound on the trader's net payment flow (or marginal valuation of the bond).